ECO-20015 - MANAGERIAL ECONOMICS II

EXERCISE 1 - MARGINS

Question 1

There was once a company that made both Nuts and Bolts in its Nuts and Bolts Factory, with both a Nuts Division and a Bolts Division. And the Factory had costs. Some costs could be easily allocated to each Division (labour and materials). But there were other costs of 100 each year called factory Overhead Costs (which came from replacing the factory guttering, rewiring, mowing the lawns around the factory and hiring security guards for the factory and so on).

Within the Factory, each year, The Nuts Division had sales of £225 and the easily allocated costs of the Division were £150, so that the Nuts Division made positive contributions to the factory budget of £75. Within the Factory, the Bolts division also had sales of £225 but had easily allocated costs of £190 so that overall the Bolts Division made positive contributions to the factory budget of a lesser £35.

Overall, given the £75 from the Nuts and the £35 from the Bolts, and remembering the Overhead Costs of £100, the Factory made a Profit of £10 (=75+35-100) to pass on to the Company Accounts. And the Company was happy that such Profits appeared.

The Company Budget Scrutineers on drawing up the Division Budgets noted that if Overhead Costs were shared 50-50 between the Divisions: (i) The Nuts Division's positive contribution was reduced from £75 to £25. (ii) The Bolts Division's positive contribution of £35 was reduced to a NEGATIVE -£15.

(a) Explain what happens if on the basis of the loss, the Bolts division is closed down (assume that the sales and costs of the Nuts division stay the same).

(b) Why is it not appropriate to make the decision close down the Bolts division even though it does not cover its allocated overhead costs.

Question 2

Consider the case of Freedonia Steel. Suppose that the price at which the Steel company can sell its steel domestically if it sells x units is

$$P(x) = 1000 - \frac{x}{250}.$$

Suppose the firm's total cost function is

$$TC(x) = 10,000,000 + 200x + \frac{x^2}{1000}.$$

(a) Find the average cost function and the minimum average cost.

(b) How much would the firm produce if it sold only domestically?

(c) How much should the firm export if it is to maximise profits? How much does it sell domestically in this case.

You may uses Calculus or the spreadsheet FREEDONIA STEEL to answer this question.

Question 3

The stadium builder in the textbook was able to sell 25 boxes at £1 million with building costs of £300,000 per box. Suppose the builder has to sell all boxes at the same price and suppose that if he were to sell a 26th box all units would sell for £950,000. Will the stadium builder make more money selling 25 or 26 boxes?

Question 4

Suppose that three divisions of a firm share a service facility. The earnings of each division are

$$y_i - \frac{y_i^2}{4} - \frac{Y}{10}$$

where y_i is the level of services received by division i and $Y = y_1 + y_2 + y_3$ is the total for all three divisions. The cost of the service facility are

$$\frac{1}{2} + \frac{Y}{5}.$$

(a) If the corporation chooses y_1 y_2 and y_3 to maximise earnings net of costs what levels of y_i will it choose?

(b) Now suppose that the firm does not charge the divisions for the use of the service facility and suppose that each individual division determines y_i independently. That is when the first division chooses y_1 it considers y_2 and y_3 as fixed. Calculate the maximising y_i for each division in this case. Is total profit higher or lower?

(c) How can the corporation improve earnings by charging each division a share of the costs? Why does this lead to an improvement?

[Note: See Chapter 14].

Question 5

The Football Association allocates tickets to the cup final between Liverpool and West Ham. There are 80,000 tickets to be allocated and the FA can set different ticket prices for different teams. A Liverpool supporter will pay £20 for a ticket. The price the FA can charge to a West Ham supporter is given by the function

$$\left(30 - \frac{W}{4000}\right)$$

where W is the number of seats allocated to West Ham supporters.

(a) Suppose the FA decides to give 40,000 tickets to each team. Calculate the price that West Ham supporters and Liverpool supporters will pay and calculate total gate receipts.

(b) Suppose the FA can charge the different sets of supporters different prices but that each West Ham supporter must pay the same price as every other West Ham supporter (likewise for Liverpool supporters). Calculate the marginal revenue for West Hamm supporters when they are allocated 40,000 tickets. [Hint: Total revenue is $W(30 - W/4000)$ and differentiate total revenue to find marginal revenue.

(c) Will taking one ticket away from a West Ham supporter and giving it to a Liverpool supporter increase or decrease overall gate receipts?

(d) If all tickets are sold, the number of Liverpool supporters will be 80,000-W and total gate receipts will be

$$\left(30 - \frac{W}{4000}\right) + 20(80000 - W).$$

Find the value of W which maximises total gate receipts. At this value of W what price do West Ham supporters pay for a ticket? What is the total gate receipts?

(e) For the value of W calculated in part (d), what is the marginal revenue from West Ham supporters. Why is this the same as the marginal revenue from Liverpool supporters?

Question 6

A lake is used by the fishermen of the local village. There are 100 villagers who either fish or farm. If B boats fish, then the catch per boat is $80/\sqrt{B}$. The cost of running the boat is £3 and the fishermen could alternatively work the land for a wage of £7. The price of fish is determined at the market in the town and is £1 no matter how many fish are caught. Fishermen will fish provided the profit they make by fishing exceeds what they can earn by working the land.

(a) The villagers will fish provided $(80/\sqrt{B}) - 3 \geq 7$. Find the number of boats B such that no more villagers will wish to take up fishing. Since every then every village earns £7, the total village in is £700.

(b) Total village income if B villagers fish and the remaining $100 - B$ farm is given by

$$B\left(\frac{80}{\sqrt{B}} - 3\right) + 7(100 - B).$$

Find the value of B that maximises total village income. What is total village income now?

(c) Explain why restricting the number of boats increases village income. How might the village restrict fishing to maximise total income?

ECO-20015 - MANAGERIAL ECONOMICS II

EXERCISE 2 - DOUBLE MARGINALISATION

Question 1

An internet service provider (ISP) sells access to the internet. It faces a inverse demand curve from consumers of the form $P(x) = 400 - x$. It buys phone line access from a telephone company (BT). The marginal costs for both the ISP and BT are zero.

(a) If BT charges the ISP a price of p per unit, how many units will the ISP choose to maximize its profits. Calculate the price it charges (as a function of p) and its maximum profit (as a function of p).

(b) Given the quantity that the ISP will provide, calculate how BT will choose the price p to maximise its profits. Calculate the profits of BT and the ISP.

(c) If BT charge a two-part tariff with a fixed fee and a fixed rate per unit, calculate the two-part tariff which would maximise BT's profits. Are consumers better or worse off?

(d) Are there other methods BT could try to use to achieve the same outcome?

Question 2

This is an example originally studied by *Cournot* (1838). There are two monopolies, one produces copper and one produces zinc. These two monopolies supply a large number of different brass produces (there is free entry and exit into the brass industry). Brass producers require one ton of copper and one ton of zinc to produce one ton of brass (these proportions are fixed and do not depend on how much brass is produced). No other inputs are needed (this is for simplicity). Let p_c be the price the brass companies must pay for a ton of copper and p_z the price for a ton of zinc. Since there are a large number of brass producers and free entry and exit, the price of brass per ton will just equal the cost of production, $p_b = p_c + p_z$. Suppose the demand for brass is

$$x = 90 - \frac{p_b}{10}.$$

Thus if x units of brass are sold, then bras companies demand exactly x tons of copper and x tons of zinc. Suppose (again for simplicity) that there are no costs of production for copper or zinc (more precisely that the marginal costs of an extra ton are always zero).

(a) The copper firm's profits are

$$p_c x = p_c \left(90 - \frac{p_b}{10}\right) = p_c \left(90 - \frac{p_c + p_z}{10}\right).$$

The copper firm will choose p_c to maximise profits taking p_z as given. Find the value of p_c (as a function of p_z which maximises profits.

(b) Repeat part (a) to find the price p_z (as a function of p_c) which maximises the zinc firm's profits.

(c) From the two equations derived in parts (a) and (b), find the values of p_c and p_z that maximise profits. Calculate the profits of the two firms.

(d) Now suppose the copper firm and the zinc firm merge to form a single monololy. The merged firm maximises profits by choosing p_c and p_z to maximise

$$(p_c + p_z)\left(90 - \frac{p_c + p_z}{10}\right).$$

Find the values of p_c and p_z that maximize the profits of the merged firm. Are the profits of the merged firm greater than the sum of the profits of the two firms when they acted separately? Are consumers (of brass) better or worse off?

(e) What is the (intuitive) explanation for your answer to part (d)?

(f) A merger is just one way for the two firms to co-ordinate on pricing policy. Are there other ways in which the monopolies can increase their profits?

ECO-20015 - MANAGERIAL ECONOMICS II

EXERCISE 3 - PRICE DISCRIMINATION

Question 1

A metropolitan authority seeks to reduce the operating deficits of its theatre. A market survey suggests there are distinct demand curves for public demand and student demand. Hence a two-tier pricing structure may well be optimal. The demand functions calculated are:

$$\begin{aligned} \textbf{Public demand}: &\quad P_P = 225 - 0.005x_P \\ \textbf{Student demand}: &\quad P_S = 125 - 0.00125x_S \end{aligned}$$

where P_P and P_S and in £s and x_P and x_S are tickets sold per season. During recent years, the operating budget has been £1.5 million per season to cover fixed costs (salaries, insurance and facility maintenance expenditures). In addition to these fixed costs, there is obviously variable ticket handling costs (insurance and security costs) of £25 per ticket. In short, the resulting total cost function in £s is $TC(x_P, x_S) = 1,500,000 + 25(x_P + x_S)$.

(a) Obtain the profit maximising prices, sales and profits under price discrimination. Compare the answers with the price, total sales and profits if discrimination is not practised.

(b) Calculate the price elasticities of demand at the profit maximising points under price discrimination. Compare these with the point price elasticity of demand if no price discrimination is practised.

(c) Give a graphical illustration of your answers to parts (a) and (b) of this question.

(d) Use you numbers to show that

$$\frac{P_P}{P_S} = \frac{1 + \frac{1}{\eta_P}}{1 + \frac{1}{\eta_S}}$$

where η_P and η_S are the relevant price elasticities. Briefly comment.

Question 2

A firm produces a good with constant marginal cost of £3 is able to engage in first-degree price discrimination. It knows the utility function of its customers and faces no legal difficulty in discriminating or preventing resale. One of its customers has a utility function

$$u(x, m) = 16x^{\frac{1}{3}} + m$$

where x is the amount of the good the customer buys and m is the money the customer has left over after the purchases (the initial wealth is large so we don't have to worry about any non-negativity constraint on m). Let m_0 denote the initial money the customer has.

(a) If the firm makes a take-it-or-leave-it offer, what offer should it make?

(b) If the firm sets an up-front fee and a per unit cost, what offer should the firm make?

(c) Compare your answers to parts (a) and (b) of the question.

Question 3

A good is sold in integer amounts. The marginal cost of production is £3 per unit. There are three customers all of whom have a utility function of the form $u_i(x) + m$ where x is the integer number of units consumed and m is the money left over. The utilities of the three customers are given by

$$u_1(0) = 0,\ \ u_1(1) = 10,\ \ u_1(2) = 18,\ \ u_1(3) = 23,\ \ u_1(4) = 25,\ \ u_1(5) = 26,\ \ \ldots$$
$$u_2(0) = 0,\ \ u_2(1) = 20,\ \ u_2(2) = 39,\ \ u_2(3) = 44,\ \ u_2(4) = 48,\ \ u_2(5) = 50,\ \ \ldots$$
$$u_3(0) = 0,\ \ u_3(1) = 30,\ \ u_3(2) = 50,\ \ u_3(3) = 60,\ \ u_3(4) = 62,\ \ u_2(5) = 61,\ \ \ldots$$

(a) Use trial and error (but with some sophistication) to show that if the firm choose a single price p for each customer, it should set $p = 19$.

(b) If the firm uses a fixed-fee pricing scheme and sets the price per unit to be £3, show that setting a fixed fee of £36 is optimal (again some trial and error is involved to find the solution). What is the firm's profit?

(c) Suppose the firm sets the per unit cost at £5. Find the best fixed fee. What is the firm's profit in this case?

ECO-20015 - MANAGERIAL ECONOMICS II

EXERCISE 4 - PERFECT COMPETITION

Question 1

A competitive firm has total cost function given by

$$TC(x) = F_1 + F_2 + 3x + \frac{x^2}{40,000}$$

where F_1 and F_2 are fixed costs. The firm can avoid paying F_2 if it produces 0, but it cannot avoid F_1. That is $TC(0) = F_1$. The efficient scale for this firm—the level of x which minimises average cost—is $x = 60,000$. The firm optimally supplies positive output for all prices above £5. Find the values of F_1 and F_2. [Hint: First find the sum $F_1 + F_2$ by using the information about the efficient scale.]

Question 2

Suppose that in a perfectly competitive industry the total cost for each firm is

$$TC(x) = 10,000,000 + 2x + \frac{x^2}{100,000}.$$

Demand is given by $D(p) = 500,000(42 - p)$.

(a) Assume that fixed costs are avoidable. Find the supply function for an individual firm.

(b) If the industry consists of six firms, what is the equilibrium price and quantities? (Ignore entry and exit).

(c) Now suppose there is free entry and exit, what is the equilibrium?

(d) Suppose that demand changes to become $\hat{D}(p) = 500,000(46 - p)$. What happens to the equilibrium (i) in the very short-run where output is fixed, (ii) in the intermediate-run when the number of firms is fixed but firms adjust their supply and (iii) in the long-run when there is free entry and exit.

ECO-20015 - MANAGERIAL ECONOMICS II

EXERCISE 5 - HIDDEN INFORMATION

Question 1

Suppose there are used cars worth every price from £1,000 to £3000 to *new* owners, with each price equally likely. Suppose that a car worth x to a new owner is worth $0.75x$ to its current owner. Suppose that there is a finite supply of cars at each price, while there is a much larger number of potential buyers (so the equilibrium price is the valuation of the potential buyer). Suppose the quality of the car is unknown to potential buyers.

(a) Calculate the equilibrium price and average quality of cars sold.

(b) Which cars are sold and which are not?

(c) Is the equilibrium efficient?

Question 2

There are two types of workers, high productivity types and low productivity types. High productivity workers can produce £40 of output per hour net of any other production costs. Low productivity workers produce only £20 per hour. Employers are initially unable to distinguish between the two types, but they know that 50% of the working population are high productivity and 50% are low productivity. Workers can undertake education. The cost of gaining some education level, x, in hourly wage equivalent is $c_H x$ for high productivity workers and $c_L x$ for low productivity workers. Hence a worker with an hourly wage of w, and an education level x, has a net payoff of $w - c_i x$, for $i = H, L$. Firms are perfectly competitive.

(a) Suppose that employers believe that if $x < x^*$ then the worker is a low productivity type and if $x \geq x^*$ then the worker is a high productivity type. Write down the *participation constraint* for the employer given his beliefs and draw the wage schedule $w(x)$ where the employer makes no profits.

(b) Suppose that high productivity workers choose an education level x_H and low productivity workers an education level x_L. Write down the *self-selection constraints* for both types (such that each type prefers the wage and education level of its type rather than the other type). Suppose that $c_H = 10$ and $c_L = 20$. Show that $x_H = x^* = 2$, $x_L = 0$ is a separating equilibrium. Are there other separating equilibria?

(c) Suppose again that $c_H = 10$ and $c_L = 20$. Find the values of x^* such that $x_H = x_L = 0$ is a pooling equilibrium (i.e. determine the value of x^* such that the high ability type will not prefer to choose the education level x^* and get the wage appropriate to high ability workers).

(d) Continuing to suppose that $c_H = 10$ and $c_L = 20$, which is the best separating equilibrium for the workers? Would they prefer a pooling equilibrium?

ECO-20015 - MANAGERIAL ECONOMICS II

EXERCISE 6 - INCENTIVES

Question 1

An entrepreneur has a project which will either make £100 million or £0. The chance of either outcome depends on the effort the entrepreneur exerts. If the entrepreneur tries hard the probability of the successful payoff is 1/10. If she doesn't try hard the probability of this outcome is 1/50. The entrepreneur is risk averse and has a utility function

$$\sqrt{w} - \text{disutility of effort}$$

where w is the amount of income in £s the entrepreneur gets to keep. Her disutility of effort is 0 if she doesn't try hard and 4 if she does try hard in £s equivalent.

(a) Assuming the entrepreneur acts alone, calculate her expected utility if she tries hard and if she does not try hard. Will the entrepreneur prefer to work hard or not?

(b) Now suppose there is a risk-neutral venture capitalist (VC) who will offer the entrepreneur a fixed up-front amount of B but expects to receive X back if the project is successful. Assume the entrepreneur will agree to a deal with VC only if the terms B and X offer at least what she can get acting on her own. Further assume that the VC can dictate whether the entrepreneur tries hard or not. What value of X should the VC specify? Calculate the value of B the VC chooses depending on whether the VC dictates that the entrepreneur works hard or not. Given the values of B you've calculated, will the VC prefer that the entrepreneur works hard or not?

(c) Now suppose that the VC cannot dictate the effort level chosen by the entrepreneur but must make sure the entrepreneur has an incentive to work hard. Let Y be the amount the entrepreneur gets when the project is successful and Z what she gets when it fails. Find the best contract the VC can offer the entrepreneur in this case (using $y = \sqrt{Y}$ and $z = \sqrt{Z}$ may be helpful).

Question 2

A salesperson can work hard or not so hard. If he works hard the probability of a sale is 0.4. If he doesn't work hard the probability of sale is 0.25. A sale generates a profit £50,000 and if there is no sale the profit is zero. The utility for the salesperson is

$$\sqrt{w} - \text{disutility of effort}$$

where w is the wage paid and the disutility of effort is 20 if he works hard and 10 if he works not so hard. The firm who employs the salesperson is risk-neutral. The reservation wage that the salesperson can get outside the firm is £8,100 with no disutility of effort and so corresponds to a reservation utility of 90. A contract can specify a wage X if a sale is made and a wage Y if no sale is made.

(a) If effort is contractible what contract should be offered to the salesperson?

(b) If effort is not contractible and he is offered a flat wage $X = Y$, what level of effort will he choose?

(c) Continuing to assume that effort is non-contractible, if the firm wanted the salesperson to work hard but not so hard, what contract should it offer? What is expected profits in this case.

(d) Repeat part (c) but assuming the firm wants to get the salesperson to work hard (using $y = \sqrt{Y}$ and $x = \sqrt{X}$ may be helpful).

(e) Given the answers to (c) and (d), what contract maximises the firm's expected profits?

ECO-20015 - MANAGERIAL ECONOMICS II

EXERCISE 7 - CREDIBILITY AND REPUTATION

Question 1

Amy and Bill play a trust game. If Bull trusts Amy, then either Amy can abuse Bill's trust and reap a larger payoff or she can treat Bill fairly and get a slightly smaller payoff. Suppose that if Amy treats Bill fairly both get a payoff of 2. However, if Amy abuses Bill's trust she gets a payoff of 4 and Bill gets a payoff of -1. If Bill doest trust Amy if the first place then they both get a payoff of 0.

(a) Draw this game in extensive form. (See figure 23.2 in the textbook for an example.)

(b) Explain why Amy's promise to treat Bill fairly is not credible. What is the outcome of the game?

(c) Suppose that Amy and Bill may be in a similar game in the future. In particular suppose there is always a probability of 0.75 that they will play the same game again next period and that payoffs from this overall game by using the expected value. Suppose further that Bill decides to trust Amy on the first round and continues to trust her as long as she treats him fairly, but will never trust her again if she abuses his trust. Suppose further that Amy treats Bill fairly on the first round and continues to do so unless she ever abuses his trust in which case she continues to abuse his trust in all subsequent situations. Show that these two strategies for Amy and Bill constitute a Nash equilibrium. Is Amy's promise to treat Bill fairly now credible?

(d) Find the probability of continuation that just makes Amy's promise credible.

(e) Reinterpret the above strategies in the language of reputation.

Question 2

In the film *A Beautiful Mind* there is a scene where the John Nash character explains the Nash equilibrium in terms of the choices of himself and some male friends in deciding which one of several girls to approach in the college bar. Suffice it to say that the explanation in the film isn't quite correct. However, the situation can nevertheless be explained as a game and the Nash equilibrium explained. To keep things simple, let's suppose there are two males, John and Kevin and three females, Alicia, Barbara and Cynthia. The film is set in the 1950s so the mores of the day dictate that the males can approach only one female. Let's suppose that all the girls are hot, but that Alicia is the most beautiful and that Barbara and Cynthia are just very cute. John and Kevin reason that if they approach Barbara or Cynthia the probability of a successful meeting is 1/3 (they are both pretty geekish) and if they approach Alicia the probability of a successful meeting is 1/5. They also reason that a successful meeting with Barbara or Cynthia is worth 60 points but a successful meeting with Alicia is worth 90 points. (Forgive the sexism here but it is only a movie.) They also reason that if they both approach the same girl the chances of success are halved. They work out their payoffs as expected values. If they both have to decide simultaneously who to approach this gives rise to the payoffs in Table 1.

		Kevin		
		Alicia	Barbara	Cynthia
John	Alicia	(9,9)	(18,20)	(18,20)
	Barbara	(20,18)	(10,10)	(20,20)
	Cynthia	(20,18)	(20,20)	(10,10)

Table 1: Strategic-Form of Nash's Game

(a) Find the Nash equilibrium of this game.

(b) What would happen if John gets to move first?

(c) What happens if instead the probability of a successful meeting with Alicia is 1/4 rather than 1/5. Recalculate the payoffs in Table 1 and look for the Nash equilibrium (in pure strategies) if the game is played simultaneously (as in part (a)) or sequentially (as in part (b)).

ECO-20015 - MANAGERIAL ECONOMICS II

SAMPLE SOLUTIONS

Exercise 2 Question 1

An internet service provider (ISP) sells access to the internet. It faces a inverse demand curve from consumers of the form $P(x) = 400 - x$. It buys phone line access from a telephone company (BT). The marginal costs for both the ISP and BT are zero.

(a) If BT charges the ISP a price of p per unit, how many units will the ISP choose to maximize its profits. Calculate the price it charges (as a function of p) and its maximum profit (as a function of p).

(b) Given the quantity that the ISP will provide, calculate how BT will choose the price p to maximise its profits. Calculate the profits of BT and the ISP.

(c) If BT charge a two-part tariff with a fixed fee and a fixed rate per unit, calculate the two-part tariff which would maximise BT's profits. Are consumers better or worse off?

(d) Are there other methods BT could try to use to achieve the same outcome?

Suggested Solution

(a) If BT charges the ISP a price of p per unit then since there are no other non-fixed costs, the marginal cost of the ISP is simply the constant price p it has to pay BT. The ISP's total revenue (TR_{ISP}) as a function of x is

$$TR_{ISP}(x) := P(x) \cdot x = (400 - x)x = 400x - x^2.$$

Hence the marginal revenue (MR_{ISP}) is given by

$$MR_{ISP}(x) := \frac{dTR_{ISP}(x)}{dx} = 400 - 2x$$

The ISP's profit is maximised when marginal revenue equals marginal cost. We therefore have that at a maximum

$$400 - 2x = p$$

or solving $x(p) = 200 - (p/2)$. We are assured that this is a maximum as the marginal revenue function is decreasing in x and marginal cost is constant. Thus for $x > 200 - (p/2)$ it will be profitable to decrease x as the loss in marginal revenue will be less than the marginal cost saved. Equally for $x < 200 - (p/2)$ an increase in x will generate more extra (marginal) revenue that the additional (marginal) cost and so will increase profits. Hence the price P charged by the ISP is

$$P(x) = 400 - \left(200 - \frac{p}{2}\right) = 200 + \frac{p}{2}.$$

The ISP's profits (π_{ISP}) is the difference between total revenue and total cost. Hence

$$\pi_{ISP} = 400x - x^2 - px = (400 - p)x - x^2.$$

Substituting in the ISP's optimum quantity $x(p) = 200 - (p/2)$ gives profit as a function of p:

$$\pi_{ISP}(p) = (400 - p)\left(200 - \frac{p}{2}\right) - \left(200 - \frac{p}{2}\right)^2.$$

Since $2(200 - (p/2)) = (400 - p)$ this can be simplified to give

$$\pi_{ISP}(p) = 2\left(200 - \frac{p}{2}\right)^2 - \left(200 - \frac{p}{2}\right)^2 = \left(200 - \frac{p}{2}\right)^2.$$

(b) Since BT has no costs, its profit is simply its total revenue. If it charges a price p and the ISP sets $x = 200 - (p/2)$, then BT's profit is $\pi_{BT} = TR_{BT} = p \cdot x$. Substituting for x gives total revenue (=profit) as a function of the price p charged:

$$TR_{BT}(p) = p \cdot x(p) = p\left(200 - \frac{p}{2}\right) = 200p - \frac{p^2}{2}.$$

The profit is maximised where marginal revenue equals marginal cost (= zero). The marginal revenue is

$$MR_{BT}(p) := \frac{dTR_{ISP}(p)}{dp} = 200 - p.$$

Setting this to zero (the marginal cost) we get that the profit maximising price is $p = 200$. (Again we are assured of a maximum by the previous reasoning that the marginal revenue is a declining function and the marginal cost is constant.) Substituting this value in, we have that BT's profits are

$$\pi_{BT} = TR_{BT} = 200^2 - \frac{(200)^2}{2} = \frac{(200)^2}{2} = \frac{40000}{2} = 20000.$$

The quantity chosen by the ISP is $x(200) = 100$ and so the price it sets is $P(100) = 300$. Its total revenue is $TR_{ISP} = 300 \times 100 = 30000$ and the total costs is $TC_{ISP} = px = 200 \times 100 = 20000$ and so the profits are $\pi_{ISP} = 10000$.

(c) BT can set the fixed cost F equal to the ISP's profit (or just less) and the ISP will still be better off trading with BT than not. Thus BT can set the fixed fee

$$F = \left(200 - \frac{p}{2}\right)^2.$$

Thus since BT has no costs its profit is given by

$$\pi_{TC}(p) = TR_{BT}(p) = F + p\left(200 - \frac{p}{2}\right).$$

Substituting in the value for F gives

$$\begin{aligned} TR_{BT}(p) &= \left(200 - \frac{p}{2}\right)^2 + p\left(200 - \frac{p}{2}\right) \\ &= \left(200 - \frac{p}{2}\right)\left(\left(200 - \frac{p}{2}\right) + p\right) \\ &= \left(200 - \frac{p}{2}\right)\left(200 + \frac{p}{2}\right) \\ &= 40000 - \left(\frac{p}{2}\right)^2. \end{aligned}$$

Choosing p to maximise this gives $p = 0$ and hence $x(0) = 200$ and $P = 200$. Total profits are $\pi_{BT}(0) = 40000$ which is greater than combined profits before. Equally the quantity consumed has increased and the price the ISP charges has decreased so consumers are better off. The fixed fee charge to the ISP is $F = \left(200 - \frac{p}{2}\right)^2 = 200^2 = 40000$.

The solution to parts (a), (b) and (c) can be illustrated in Figure 1. The red line is the ISP's demand. The dark blue line is the ISP's marginal revenue curve. Its marginal cost is just given by the price p that BT charges. The demand curve facing BT is therefore (again) the dark blue line as this gives where the ISP will supply for each given p. Facing this demand from the ISP, BT's marginal revenue curve is the light blue line. Since the marginal cost for BT is zero, the optimum for BT is to set quantity and price where the light blue line cuts the horizontal axis. This corresponds to a quantity of 100 and a price charged to the ISP of 200 on the dark blue line. The ISP thus charges a price P of 300 on the red line. The profits of the ISP are given by the boxed area "A" (= 100 × 100) and the profits of BT by the boxed area "B" (= 100 × 200).

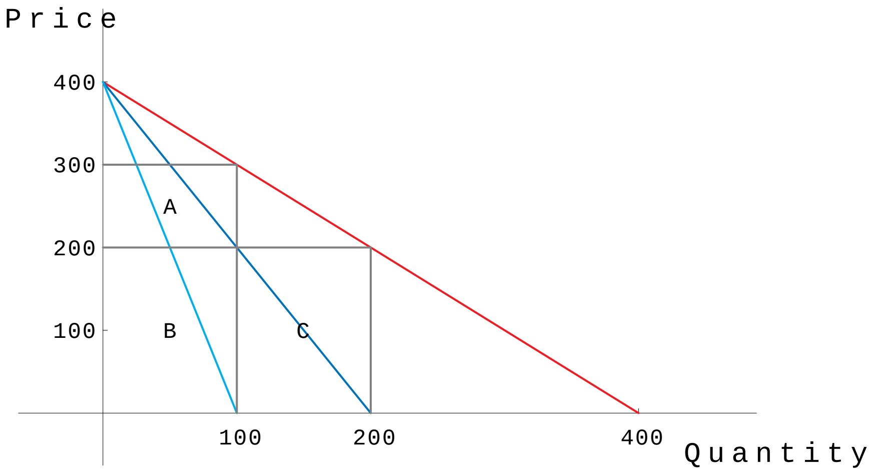

Figure 1: The BT-ISP Solution

If BT sets a fixed fee F it can set $p = 0$. Thus the ISP produces where the dark blue line cuts the horizontal axis, that is $x = 200$ and the price is determined by the red line so $P = 200$. The total profits is the boxed area "B+C" (= 200 × 200) and BT sets the fixed fee to exactly this area.

(d) Two possibilities would be (i) for BT to sell directly rather than through the ISP and (ii) would be to vertically integrate to buy the ISP and run operations. In both cases there would then be only one relevant marginal revenue curve given by the dark blue line. In both cases this would produce the identical solution provided there were no additional costs of selling directly or no extra costs incurred by operating an integrated firm rather than two separate ones.

Exercise 5 Question 1

Suppose there are used cars worth every price from £1,000 to £3000 to *new* owners, with each price equally likely. Suppose that a car worth x to a new owner is worth $0.75x$ to its current owner. Suppose that there is a finite supply of cars at each price, while there is a much larger number of potential buyers (so the equilibrium price is the valuation of the potential buyer). Suppose the quality of the car is unknown to potential buyers.

(a) Calculate the equilibrium price and average quality of cars sold.

(b) Which cars are sold and which are not?

(c) Is the equilibrium efficient?

Suggested Solution

(a) The current owner will be prepared to sell provided the price at which is sold is greater than the value to him. Let p be the price at which cars sell. A seller will sell a car of value x provided $p \geq 0.75x$. Buyers however, do not know the value of cars and must calculate the expected quality of cars on the market. Buyers can reason that the only cars on the market are those such that $p \geq 0.75x$ or $x \leq \frac{4}{3}p$. Since the lowest quality of car has a value of 1,000 and values are uniformly distributed, the average quality of cars on the market as a function of p is given by

$$\mu(p) = \frac{1}{2}\left(1000 + \frac{4}{3}p\right) = 500 + \frac{2}{3}p.$$

Buyers will buy only if the price they pay is less than the expected value of the car they will be buying, $p \leq 500 + \frac{2}{3}p$. This is illustrated in Figure 2 which draws the density function for car value. For a given value of p only cars in the shaded area will be offered for sale. The expected value of such cars is exactly at the mid-point between 1000 and $\frac{4}{3}p$, that is at $500 + \frac{2}{3}p$. Since there are many more buyers and sellers, the price will be bid up exactly to this value and hence the equilibrium price is found by solving

$$p = 500 + \frac{2}{3}p.$$

Solving this equation gives the equilibrium value $p = 1500$. Hence the highest value car offered for sale will have a value of $\frac{4}{3} \times 1500 = 2000$ and the average value of car offered for sale is worth 1500 to buyers.

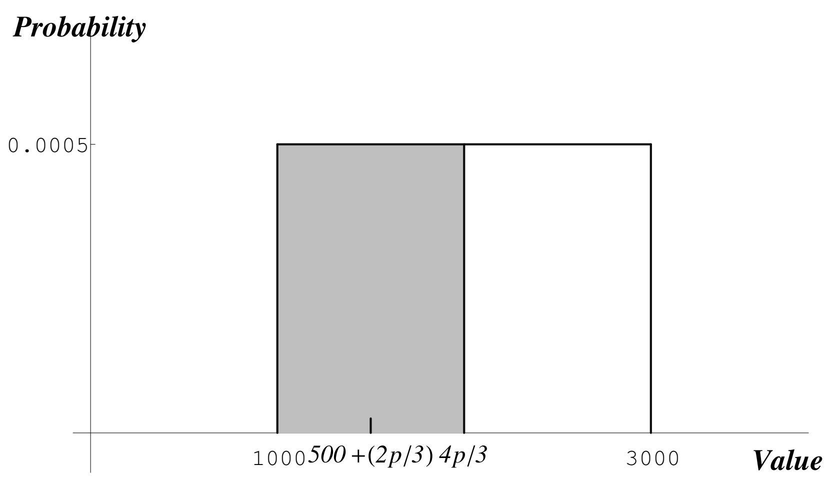

Figure 2: Expected Car Quality

It can be checked that this is the equilibrium price. For example if the equilibrium price were 1800 say then cars of vale up to 2400 would be offered for sale. In this case the average quality of the cars on sale would be 1700 and buyers would not be prepared to pay 1800 for cars which have an expected value of $\frac{1}{2}(1000 + 2400) = 1700$. Thus the price must fall. Likewise if the price were 1200 say, then cars of value up to 1600 would be offered for sale with an average quality of $\frac{1}{2}(1000 + 1600) = 1300$. Buyers would be keen to buy for 1200 cars they expected to be of value 1300 and the price would be bid up.

(b) Only the low quality cars worth less than 2000 to buyers are offered for sale. These cars are bought and higher quality cars are not sold.

(c) It is efficient that all cars are sold as buyers value them more than sellers. However, there is an *adverse selection* problem. As the price falls, only lower quality cars are offered for sale and therefore the expected value of cars on offer falls leading buyers to offer lower prices which in turn leads to lower quality cars being offered for sale and so on. This inefficiency in the market is entirely due to the hidden information problem as no participant (buyer or seller) has any market power. Here price performs a dual role of providing information about quality and equilibrating the market.

Exercise 6 Question 1

An entrepreneur has a project which will either make £100 million or £0. The chance of either outcome depends on the effort the entrepreneur exerts. If the entrepreneur tries hard the probability of the successful payoff is 1/10. If she doesn't try hard the probability of this outcome is 1/50. The entrepreneur is risk averse and has a utility function

$$\sqrt{w} - \text{disutility of effort}$$

where w is the amount of income in £s the entrepreneur gets to keep. Her disutility of effort is 0 if she doesn't try hard and 4 if she does try hard in £s equivalent.

(a) Assuming the entrepreneur acts alone, calculate her expected utility if she tries hard and if she does not try hard. Will the entrepreneur prefer to work hard or not?

(b) Now suppose there is a risk-neutral venture capitalist (VC) who will offer the entrepreneur a fixed up-front amount of B but expects to receive X back if the project is successful. Assume the entrepreneur will agree to a deal with VC only if the terms B and X offer at least what she can get acting on her own. Further assume that the VC can dictate whether the entrepreneur tries hard or not. What value of X should the VC specify? Calculate the value of B the VC chooses depending on whether the VC dictates that the entrepreneur works hard or not. Given the values of B you've calculated, will the VC prefer that the entrepreneur works hard or not?

(c) Now suppose that the VC cannot dictate the effort level chosen by the entrepreneur but must make sure the entrepreneur has an incentive to work hard. Let Y be the amount the entrepreneur gets when the project is successful and Z what she gets when it fails. Find the best contract the VC can offer the entrepreneur in this case (using $y = \sqrt{Y}$ and $z = \sqrt{Z}$ may be helpful).

Suggested Solution

(a) If she acts alone and tries hard her expected utility (EU) is

$$\text{EU} = \frac{1}{10}\left(\sqrt{100,000,000} - 4\right) + \frac{9}{10}(0 - 4) = 996.$$

If she doesn't try hard her expected utility is

$$\text{EU} = \frac{1}{50}\left(\sqrt{100,000,000} - 0\right) + \frac{49}{50}(0 - 0) = 200.$$

Thus she would prefer to work hard if working alone as the expected utility is greater.

(b) As the VC is risk-neutral and can dictate the entrepreneurs's effort level, the VC should bear all the risk and the entrepreneur should get a constant payment. Thus the VC should pay an up-front fee of B and wholly retain the £100m return if the project is successful. In any other arrangement the VC would be able to increase its expected return as the risk-averse entrepreneur will be prepared to pay a risk premium to reduce the risk That is the entrepreneur would accept a sure return of something less than the expected value of the risk and this increases the VC's expected payoff. Since the entrepreneur then gets B for sure if the entrepreneur is to accept and work hard, then B must satisfy

$$\sqrt{B} - 4 \geq 996$$

or $\geq 1,000,000$ where the 996 reservation utility is what the entrepreneur can get by going it alone (when she prefers to work hard). If the entrepreneur is not to work hard then B must satisfy

$$\sqrt{B} - 0 \geq 996$$

or $B \geq 992,016$. In both cases there is not need for the VC to pay an up-front payment that delivers more than the utility from the entrepreneur going it alone. Thus we have $B = 1,000,000$ when the VC dictates hard effort and $B = 992,016$ when it dictates low effort. The VC's profits if it dictates high effort is the expected return from the investment less the up-front payment, that is

$$\frac{1}{10}100,000,000 - 1,000,000 = 9,000,000.$$

On the other hand if the VC dictates low effort the VC's expected profit is (since the probability of success is lower)

$$\frac{1}{50}100,000,000 - 992,016 = 1,007,984.$$

Thus the VC will prefer the entrepreneur to work hard because his/her expected payoff is much higher.

(c) Suppose the VC decides he/she wants the entrepreneur to work hard. If the VC gives the entrepreneur Y if the project is successful and Z if it is not then the entrepreneur's expected utility will be

$$\text{EU}_{hard} = \frac{1}{10}\left(\sqrt{Y} - 4\right) + \frac{9}{10}\left(\sqrt{Z} - 4\right).$$

The VC must make sure of two things. First the VC must satisfy the *participation constraint* for the entrepreneur to prefer to work with the VC rather than work alone. Thus Y and Z must satisfy

$$\frac{1}{10}\left(\sqrt{Y} - 4\right) + \frac{9}{10}\left(\sqrt{Z} - 4\right) \geq 996.$$

Secondly Y and Z must satisfy an incentive constraint that the entrepreneur prefers to work hard rather than not. The expected utility of not working hard is

$$\text{EU}_{not} = \frac{1}{50}\left(\sqrt{Y} - 0\right) + \frac{49}{50}\left(\sqrt{Z} - 0\right).$$

Thus the constraint to be satisfied is $\text{EU}_{hard} \geq \text{EU}_{not}$ or

$$\frac{1}{10}\left(\sqrt{Y} - 4\right) + \frac{9}{10}\left(\sqrt{Z} - 4\right) \geq \frac{1}{50}\left(\sqrt{Y} - 0\right) + \frac{49}{50}\left(\sqrt{Z} - 0\right).$$

Replacing $\sqrt{Y}$ with y and $\sqrt{Z}$ with z the two equations to be satisfied can be rewritten as

$$\frac{1}{10}y + \frac{9}{10}z \geq 1000$$

and

$$\frac{4}{50}(y-z) \geq 4.$$

These two inequalities must be satisfied as equalities in an optimum contract as it will always be better for the VC to reduce the risk imposed on the entrepreneur as possible and thus make the difference between y and z as small as possible and it will never be optimal to give the entrepreneur more utility than his/her next best alternative. Solving these two equations gives $y = 1045$ and $z = 995$. Therefore the entrepreneur will be left with $Z = 995^2 = 990,025$ if the project fails. That is the value of B, the up-front payment. Similarly if the project succeeds, the entrepreneur gets $Y = 1045^2 = 1,092,025$. Since the entrepreneur gets B in any case the amount the VC gives the entrepreneur in the event of success is $X = Y - B = 1,092,025 - 990,025 = 102,000$. The VC gets the rest, that is $100,000,000 - 102,000 = 99,898,000$. The VC's expected profit is therefore

$$\frac{1}{10}(100,000,000-X)-B = \frac{1}{10}(100,000,000-102,000)-990,025 = 8,999,775.$$

If the VC wanted the entrepreneur not to work hard, then the best contract to offer provide the entrepreneur with a flat fee of B which satisfies the participation constraint

$$\frac{1}{50}\left(\sqrt{B}-0\right)+\frac{49}{50}\left(\sqrt{B}-0\right) = \sqrt{B} \geq 996.$$

This gives a upfront payment $B = 992,016$ and so the VC's expected profit is

$$\frac{1}{50}(100,000,000) - 992,016 = 1,007,984$$

which is less than the profit from getting the entrepreneur to take the hard effort. Thus getting the entrepreneur to work hard, paying an upfront fee of $B = 990,025$ and a pay-back of $X = 102,000$ if successful is the optimum incentive contract for the VC.