ECO-30004 - OPTIONS AND FUTURES

EXERCISE 3

1. For a given strike price, briefly explain why the prices of the American puts and calls rise with the length of time until maturity.

2. Consider two European put options on stock ABC with strike prices 10 and 20 and the same maturity date T. The prices of the two options today (at date $t < T$) are $p(10)$ and $p(20)$ respectively. Briefly explain why $p(20) > p(10)$. Consider a **bear spread** with puts by writing the put option with the strike price of 10 and buying the put with the strike price of 20. Detail the cash flows now at date t and the cash flows at the maturity date T. Explain why there would be an arbitrage opportunity if $p(10) > p(20)$.

3. Consider a one period binomial model. The stock price is initially 100 and rises to either 120 or 180 (in this case $u = 0.8$ and $d = 0.2$). Suppose that the rate of interest is 40% and that both the changes in price are equally likely (that is, $r = 0.4$ and $\pi = 1/2$). Suppose there is a put option with a strike price of 141 that expires after one period.

   (a) Calculate the expected return on the stock, the volatility (standard deviation) and the risk premium.

   (b) How can you create a synthetic put option with a strike price of 141 using only the underlying stock and the risk-free asset?

   (c) Use the synthetic portfolio to determine the price of the put option.

   (d) Calculate the risk-neutral probability in this example. Use the risk-neutral probability to price the put option.

   (e) Calculate the value of Δ required to create a riskless portfolio of the stock and the put option. How can the put option be priced using this approach?

   (f) Compute the risk premium on the put option. Calculate the standard deviation of the put return.

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EXERCISE 3 - Outline Solutions

1. The longer dated option has all the same opportunities to exercise for profit as the shorter dated option and more besides. That the longer dated option has the possibility to be exercised for a profit after the shorter dated option has expired. Provided that there is a chance that the option will end up in the money some time between the two maturity dates these extra opportunities will be valuable and the longer dated option will trade at a higher price.

2. The put option gives the right to sell. The right to sell at a higher price will be more valuable. That is if the stock price is between 10 and 20 the higher strike price option can be exercised for a profit of $20 - S_t$ whereas the lower strike price option cannot be exercised for profit. Thus provided there is a positive probability of the underlying being between 10 and 20 the higher strike price option will trade at a higher price.

   The bear spread with puts gives a current cash flow of $p(10) - p(20)$. The cash flows at maturity are given in the Table. Since the cash flows are always non-negative, if the current cash flow were also positive, $p(10) > p(20)$, there would be an arbitrage opportunity generating current cash inflows and no cash out flows at maturity.

Position	$S_T < 10$	$10 < S_T < 20$	$20 < S_T$
Short put	$-(10 - S_T)$	0	0
Long put	$20 - S_T$	$20 - S_T$	0
Overall	10	$20 - S_T > 0$	0

Table 1: Bear Spread with Puts

3. (a) The expected return is 50%, the volatility 40% and the risk premium 10%.

   (b) We buy Δ units of the stock and invest B in the risk free asset. The put option is valueless in the up state and can be exercised for a profit of $141 - 120 - 21$ in the down state. Thus we solve the two equations $180\Delta + \frac{7}{5}B = 0$ and $120\Delta + \frac{7}{5}B = 21$. This gives $\Delta = -\frac{7}{20}$ and $B = 45$. That is we short sell a fraction 7/20 of the stock and invest 45 in the risk-free asset.

(c) The put price is equal to the value of the synthetic which is $-\frac{7}{20}100 + 45 = -35 + 45 = 10$.

(d) The risk neutral probabilities satisfy the equation

$$\frac{180\rho + 120(1-\rho)}{1+\frac{2}{5}} = 100.$$

Solving gives $\rho = \frac{1}{3}$. Hence the value of the put is

$$p = \frac{(1-\rho)21}{1+r} = \frac{14}{1.4} = 10.$$

(e) Buying Δ units of the stock and the put option gives a portfolio worth 180Δ in the up state and $120\Delta + 21$ in the down state. Equating gives $\Delta = \frac{7}{20}$ and a risk-free payoff of 63. The present value of this payoff is $63/1.4 = 45$. The portfolio of Δ units of the stock and the put has a value of $100\Delta + p = 35 + p$ and since this is equal to 45 we can deduce that $p = 10$.

(f) The return on the put is -100% in the up state and 110% in the down state. Thus the expected return is 5%, the volatility is 105% and the risk premium is -5%.