

Chapter 1

An Overview of Derivative Securities

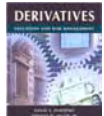
- A derivative is an instrument whose value depends on the value of an underlying variable.
 - Interest rate or foreign exchange rate
 - Index value such as a stock index value
 - Commodity price
 - Common stock (NEW! – Single Stock Futures)
 - Other
- Thus, the value of a derivative is ‘derived’ from another variable.



The Four Basic Derivatives

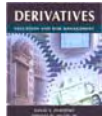
- Forward Contracts
- Futures Contracts
- Swaps
- Options

However, many say there are only TWO basic derivatives: forwards and options.



Derivative Markets

- Exchange-traded futures and options
 - standardized products
 - trading floor or computerized trading
 - virtually no credit risk
- Over-the-Counter forwards, options, & swaps
 - often non-standard (customized) products
 - telephone (dealer) market
 - some credit risk



Uses of Derivatives

- To hedge or insure risks; i.e., shift risk.
- To reflect a view on the future direction of the market, i.e., to speculate.
- To lock in an arbitrage profit
- To change the nature of an asset or liability.
- To change the nature of an investment without incurring the costs of selling one portfolio and buying another.



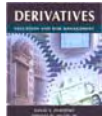
The Importance of this Subject - I

- The sheer magnitude of size of this market, in itself, makes this an interesting topic.
- Firms and individuals face financial risks that are greater than ever.
- Derivative markets allow participants to shed, or hedge, risk.
- Derivative markets allow participants to take risk, i.e., speculate.
- Using derivatives allows individuals and firms to create payoff patterns that are compatible with their beliefs and degree of risk aversion, at a low cost.



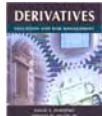
The Importance of this Subject - II

- You will understand the vital linkage between the **underlying** market (aka the **spot** market or the **cash** market) and the derivative market.
- You will see that the price of the derivative depends on the price of the underlying asset.
- You will see what caused many derivative “disasters” of recent times: i.e., Baring’s Bank, Sumitomo Bank, Long Term Capital Management.
- You will avoid derivative disasters:
 - you will learn to avoid using derivatives when you do not understand their risks
 - you will learn how to use derivatives correctly because you will understand their risks and rewards.



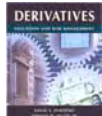
Just How Big is the Derivatives Market?

- The Bank for International Settlements (BIS) estimated the total notional principal of outstanding OTC derivatives to be \$111 *trillion*, as of December 2001! (<http://www.bis.org/statistics/index.htm>)
- However, the figure above overstates the true size of the market.
 - notional principal
 - netting
- 4.3 billion derivative contracts traded on global organized exchanges in 2001 (in contrast, NYSE volume in 2002 is running at a 322 billion shares per year rate, as of August 1, 2002).



Forward Contracts - I

- A forward contract gives the owner the **right** and **obligation** to buy a specified asset on a specified date at a specified price.
- The seller of the forward contract has the **right** and **obligation** to sell the asset on the date for the price.
- At the end of the forward contract, at “delivery,” ownership of the good is transferred and payment is made from the purchaser to the seller.



Forward Contracts - II

- Generally, no money changes hands on the origination date of the forward contract.
- However, collateral may be demanded.
- **Delivery options** may exist concerning
 - the quality of the asset
 - the quantity of the asset
 - the delivery date
 - the delivery location.
- If your position has value, you face the risk that your counterparty will **default**.

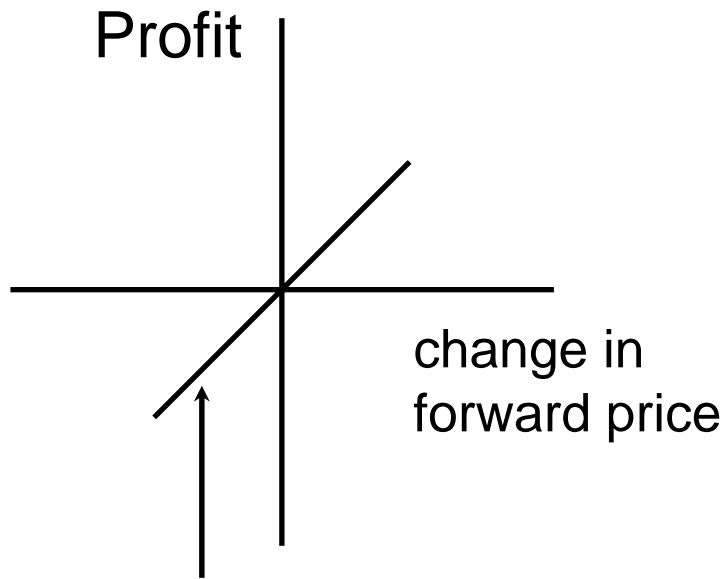


Forward Contracts - III

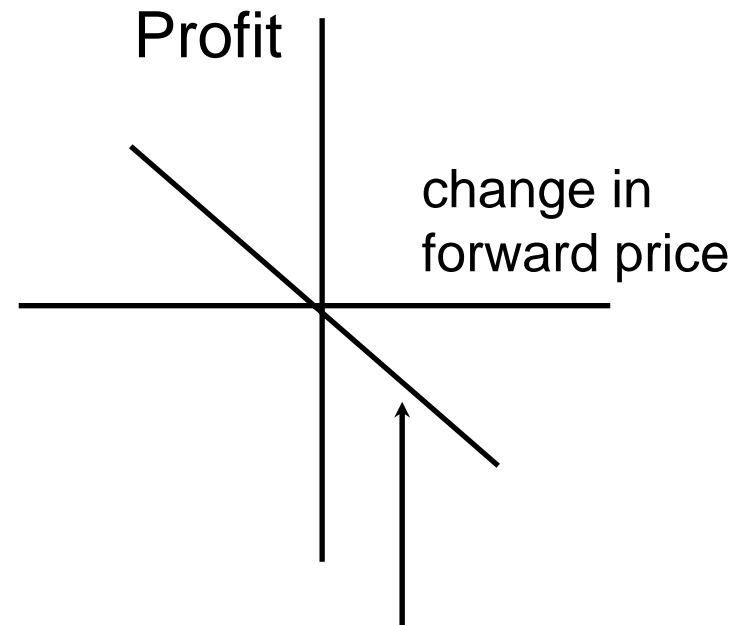
- On August 27, 2002, you enter into an agreement to buy £1 million in six months at a forward exchange rate of \$1.5028/ £. The spot exchange rate is \$1.5250/£.
- This obligates the trader to pay \$1,502,800 for £1 million on February 27, 2003.
- What happens if the spot exchange rate on February 27, 2003 is \$1.60/£? \$1.40/£?



Forward Contracts - IV



Profit Diagram for a **long** forward position



Profit Diagram for a **short** forward position



Futures vs. Forwards - I

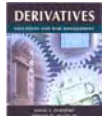
- Futures are similar to forwards, **except:**
 - Futures trade on futures exchanges (CME, CBOT, LIFFE, etc.).
 - Futures are standardized contracts
 - this increases their liquidity
 - but sometimes firms prefer precise, custom made (OTC) forward contracts

More =>



Futures vs. Forwards - II

- Default risk for futures is lower because:
 - The clearinghouse of the exchange guarantees payments.
 - An initial margin is required.
 - Futures contracts are “**marked to market**” daily (daily resettlement)
- A consequence of daily resettlement is that futures contract are like a portfolio of forward contracts, each with a delivery day one day later.



2001 Trading Volume: Top 10 Global Futures Exchanges

Exchange	Number of Contracts
Eurex (Germany & Switzerland)	435,141,707
Chicago Mercantile Exchange (U.S.)	315,971,885
Chicago Board of Trade (U.S.)	209,988,002
LIFFE (UK)	161,522,775
BM&F (Brazil)	94,174,452
NY Mercantile Exchange (U.S.)	85,039,984
Tokyo Commodity Exchange (Japan)	56,538,245
London Metal Exchange (UK)	56,224,495
Paris Bourse SA (France)	42,042,673
Sydney Futures Exchange (Australia)	34,075,508



Swaps

- A swap contract obligates two parties to exchange, or swap, cash flows at specified future dates.
- A swap is like a portfolio of forwards. Each forward in a swap has a different delivery date, and the same forward price.
- It's a \$60+ trillion market, as of Dec., 2001 (in terms of notional principal)



An Example of a 'Plain Vanilla' Interest Rate Swap

- I agree to pay you 8% of \$40 million each year for the next five years.
- You agree to pay me whatever 1-year LIBOR is (times \$40 million) for each of the next five years.
- \$40 million is the **notional principal**.
- If LIBOR > 8%, you pay me: $(\text{LIBOR} - 8\%) * \$40 \text{ million}$
- If LIBOR < 8%, I pay you: $(8\% - \text{LIBOR}) * \$40 \text{ million}$
- I am **long** five Forward Rate Agreements (FRAs), with delivery dates at the end of each of the next five years.
- You are **short** five Forward Rate Agreements (FRAs), with delivery dates at the end of each of the next five years.



Swaps

- Credit risk only involves the Present Value of the remaining expected cash flows from the swap (~3.5% of notional principal).
- Credit risk does **NOT** involve the notional principal.
- Only the party that expects to be paid the remaining **net** cash flows faces current default risk.

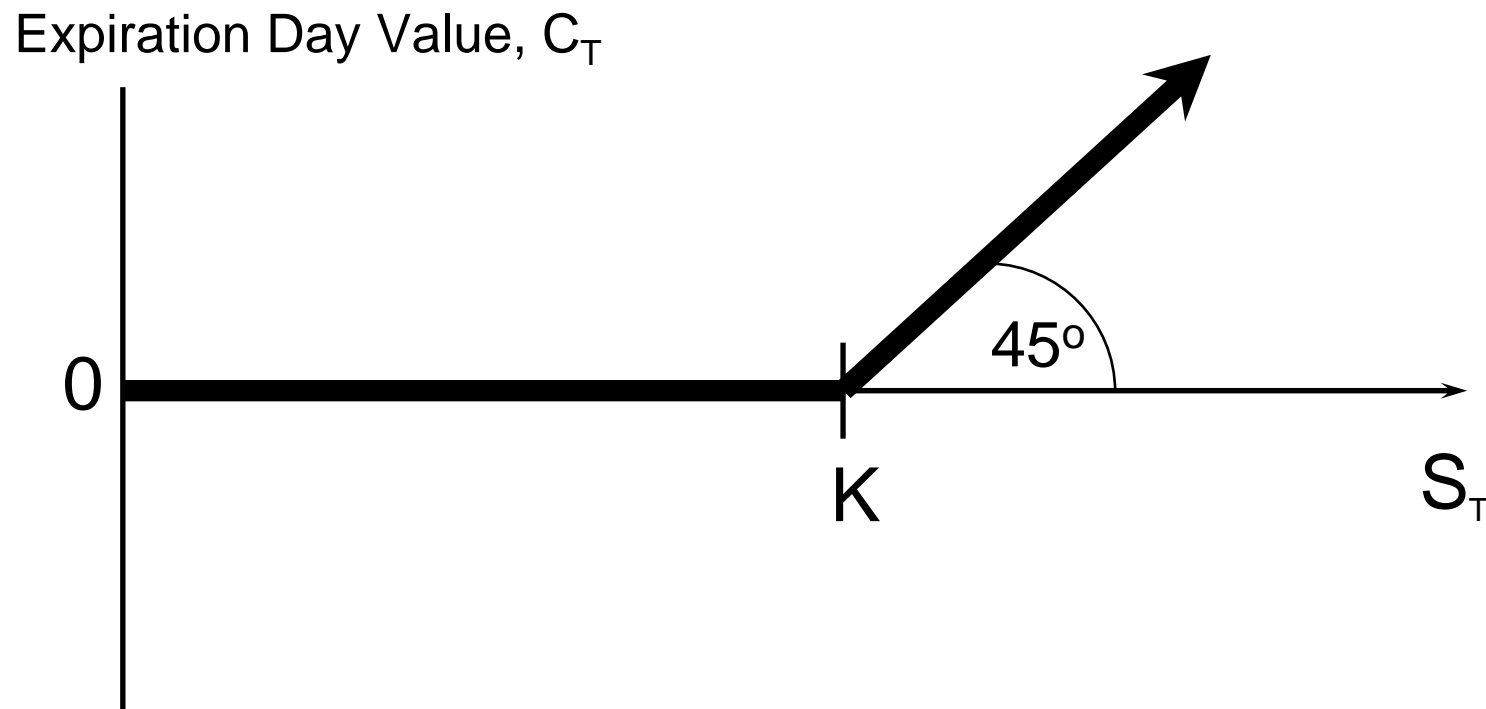


Call Options

- A call option is a contract that gives the **owner** of the call option the **right, but not the obligation, to buy** an underlying asset, at a fixed price ($\$K$), on (or sometimes before) a pre-specified day, which is known as the expiration day.
- The **seller** of a call option, the call **writer**, is **obligated** to deliver, or **sell**, the underlying asset at a fixed price, on (or sometimes before) expiration day (T).
- The fixed price, K , is called the *strike price*, or the *exercise price*.
- ***Because they separate rights from obligations, call options have value.***

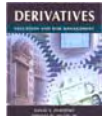
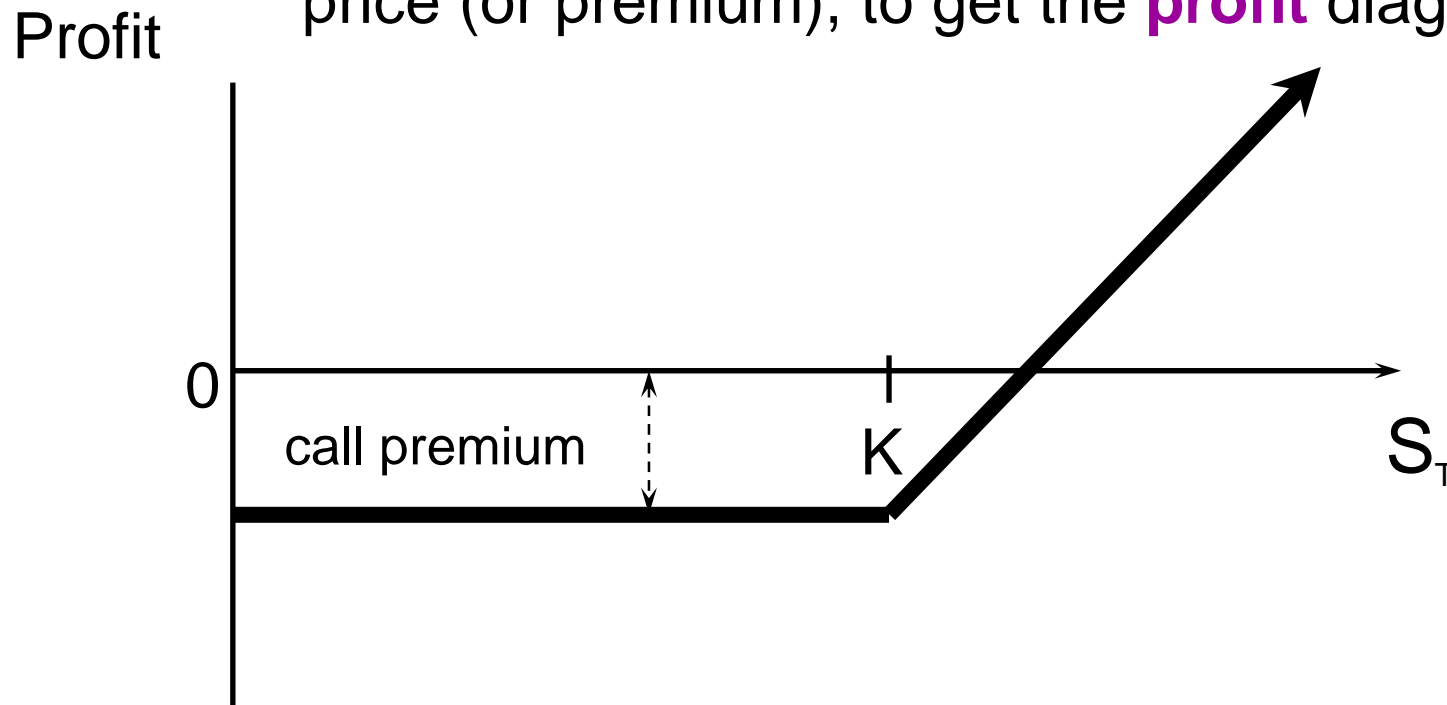


Payoff Diagram for a Long Call Position, at Expiration

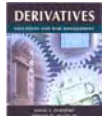
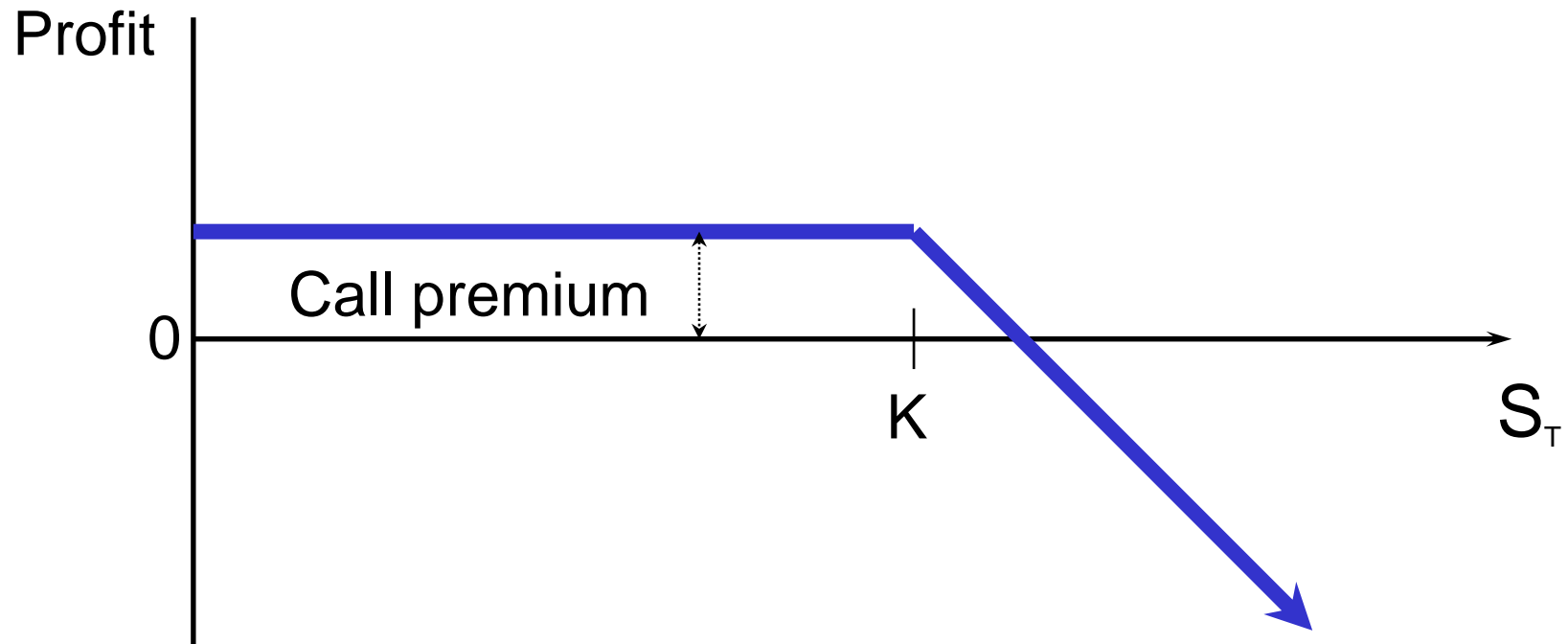


Profit Diagram for a Long Call Position, at Expiration

We lower the **payoff** diagram by the call price (or premium), to get the **profit** diagram

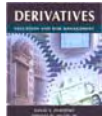


Profit Diagram for a Short Call Position, at Expiration

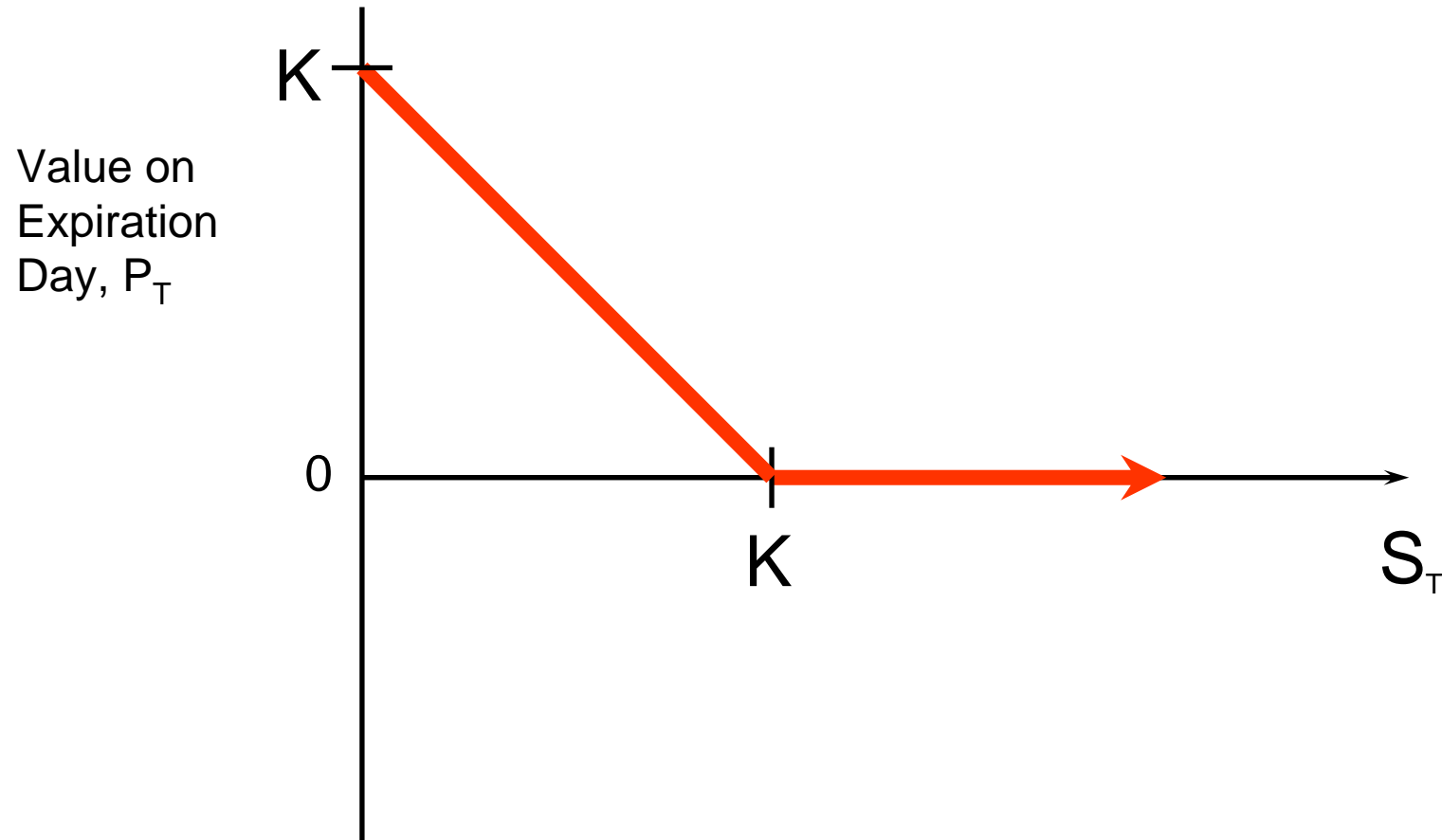


Put Option Contracts

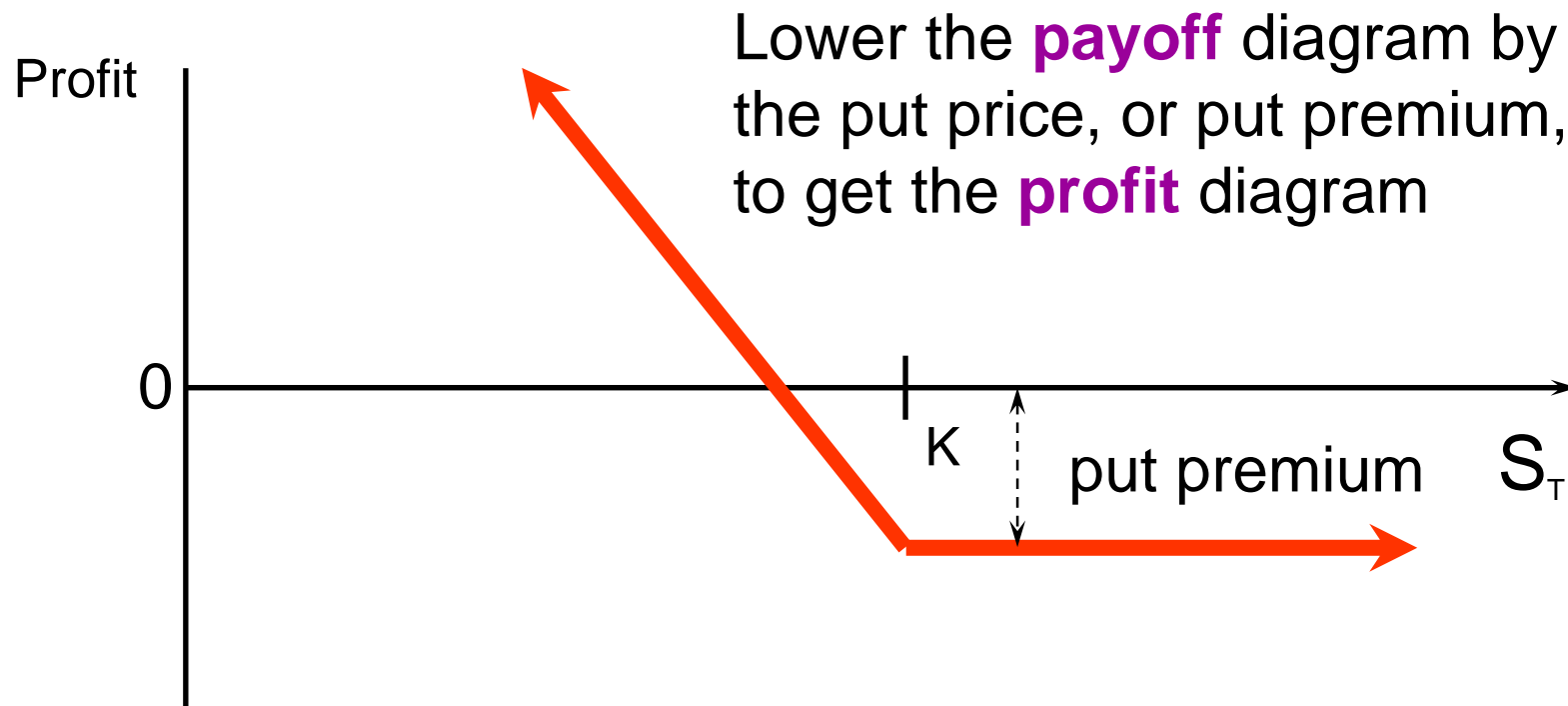
- A put option is a contract that gives the **owner** of the put option the **right, but not the obligation, to sell** an underlying asset, at a fixed price, on (or sometimes before) a pre-specified day, which is known as the expiration day (T).
- The **seller** of a put option, the put **writer**, is **obligated** to take delivery, or **buy**, the underlying asset at a fixed price (\$K), on (or sometimes before) expiration day.
- The fixed price, K, is called the *strike price*, or the *exercise price*.
- ***Because they separate rights from obligations, put options have value.***



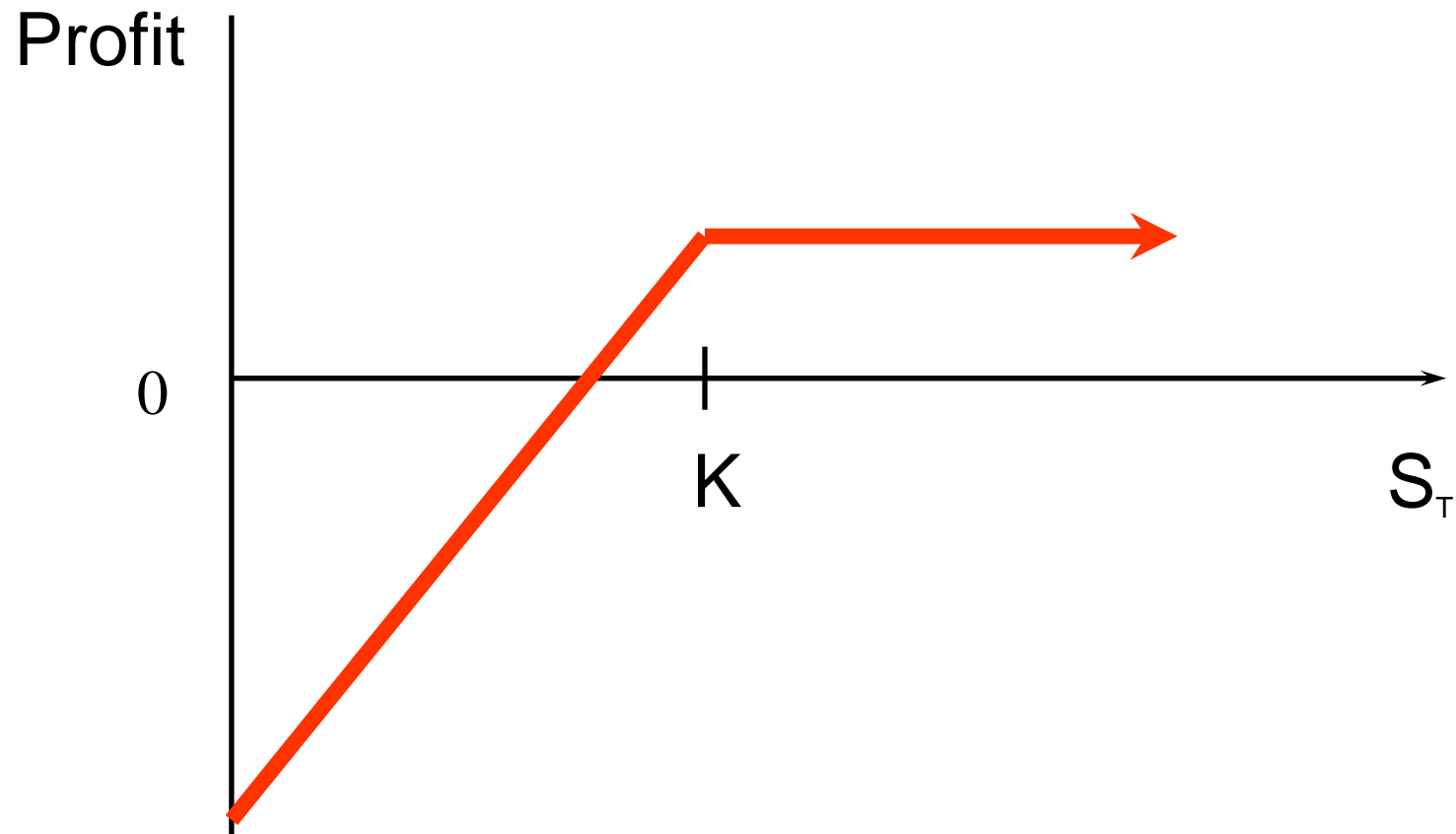
Payoff diagram for a long put position, at expiration



Profit Diagram for a Long Put Position, at Expiration



Profit Diagram for a Short Put Position, at Expiration



Option Exchanges

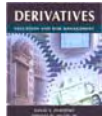
U.S. Option Exchange	Options Traded in 2001
Chicago Board Options Exchange	306,667,851
American Stock Exchange	205,103,884
Pacific Stock Exchange	102,701,752
Philadelphia Stock Exchange	101,278,870

- 1.4 billion options traded, globally, on exchanges in 2001.
- Trading on the International Securities Exchange commenced in 2000. It was the first new US securities exchange in 27 years. All trading is electronic. 77.5 million contracts traded on the ISE in the six months ending Aug. 15, 2002.
- Other leading international option exchanges include OM Stockholm (Sweden), Euronext Amsterdam (Netherlands), etc.



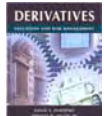
Types of Derivative Traders

- Hedgers (Risk Shifters)
- Speculators (Risk Takers)
- Arbitraguers (Alchemists)
- Note Bene: Some of the large trading losses in derivatives occurred because individuals who had a mandate to hedge risks switched to being speculators



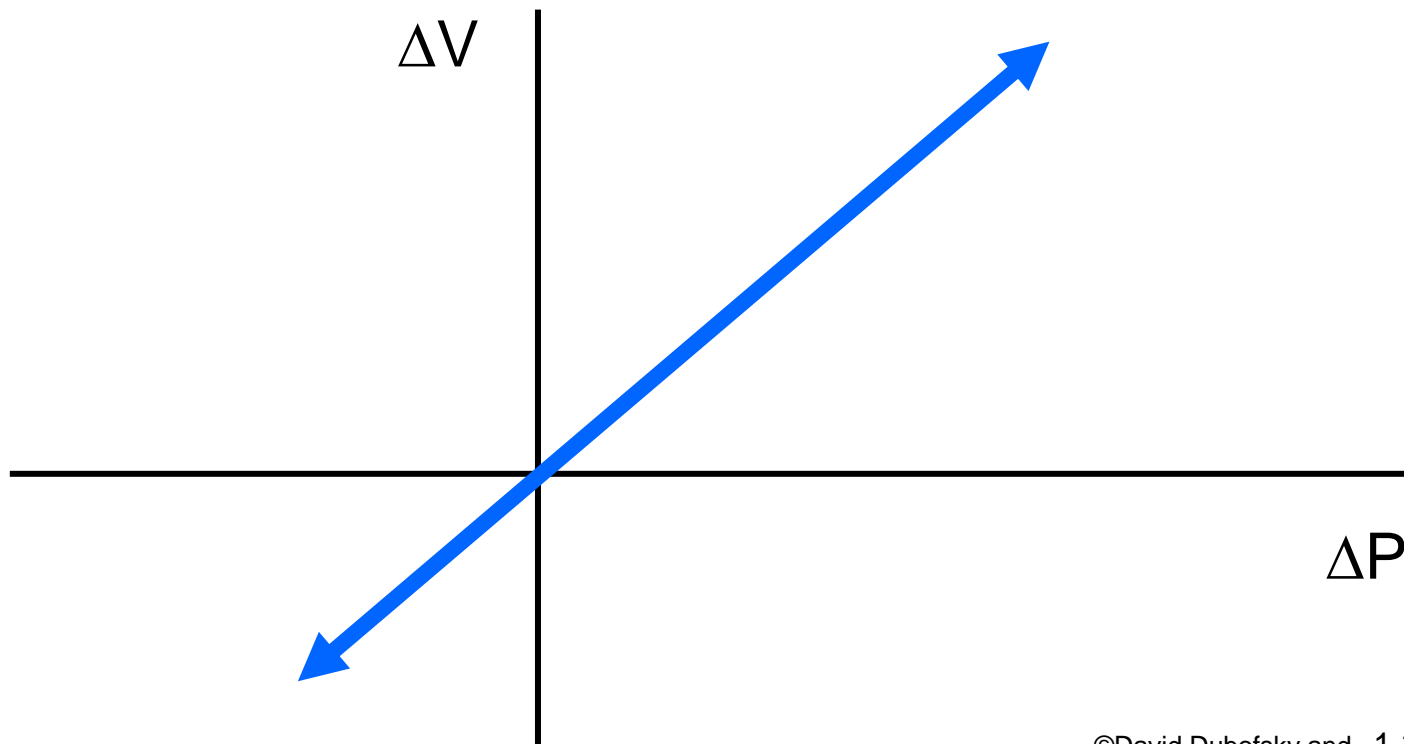
Some Extra Slides on this Material

- Note: In some chapters, we try to include some extra slides in an effort to allow for a deeper treatment of the material in the chapter.
- If you have created some slides that you would like to share with the community of educators that use our book, please send them to us!

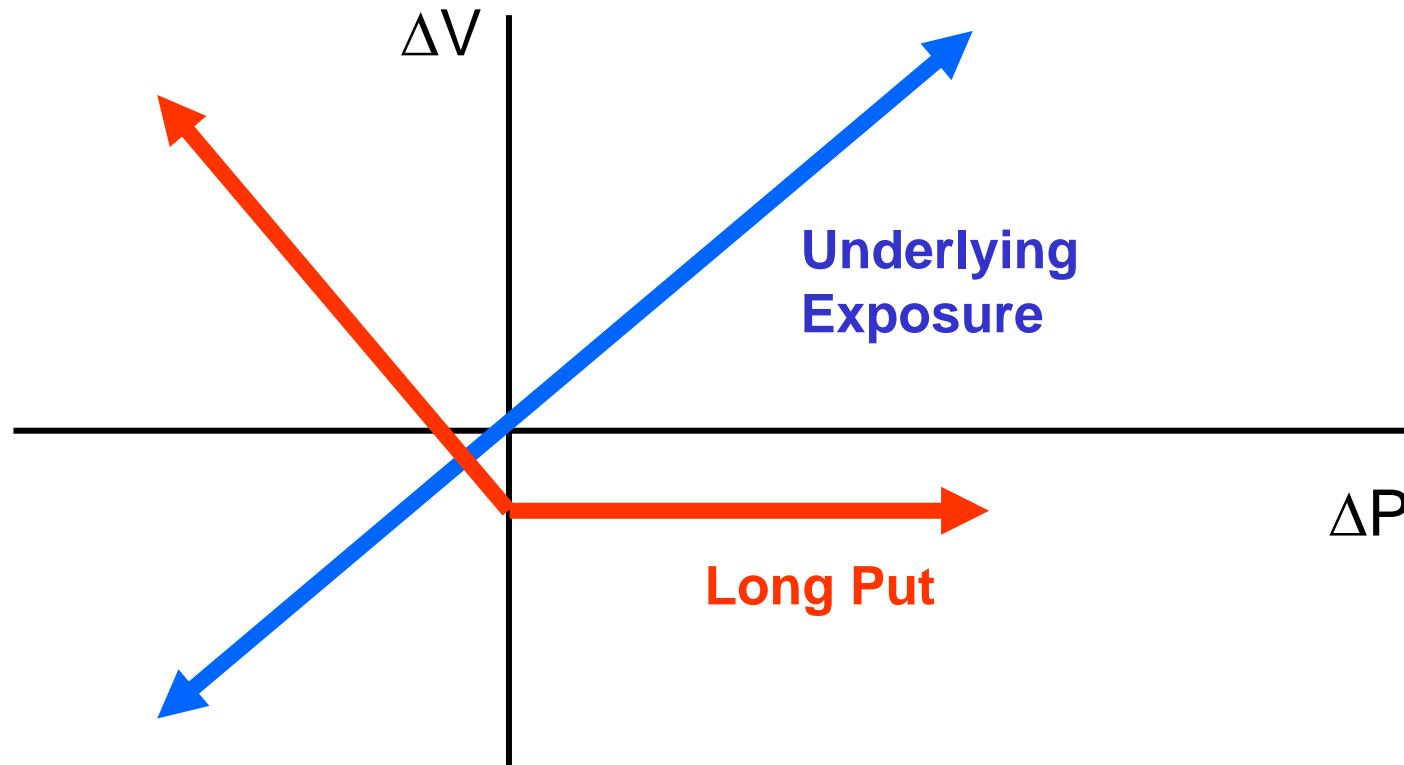


Using Options in Risk Management, I: Options Provide Insurance

Suppose a firm's value is 'exposed'
to the risk that some price will **decrease**



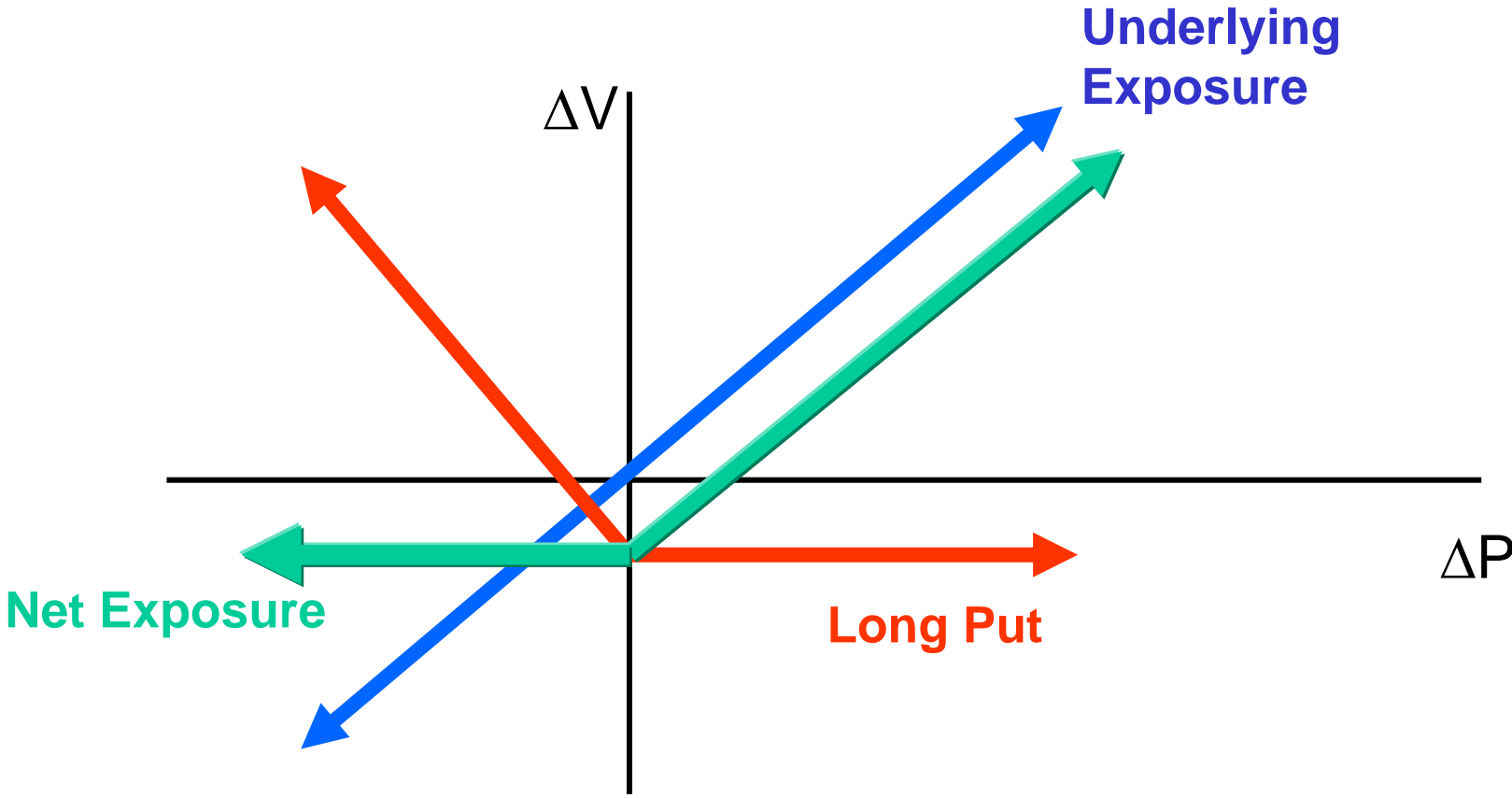
Suppose the Firm Buys An at-the-money Put Option



Recall: The firm is exposed to the risk that a price will decline



This is the Resulting "Net" Exposure

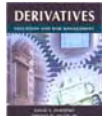
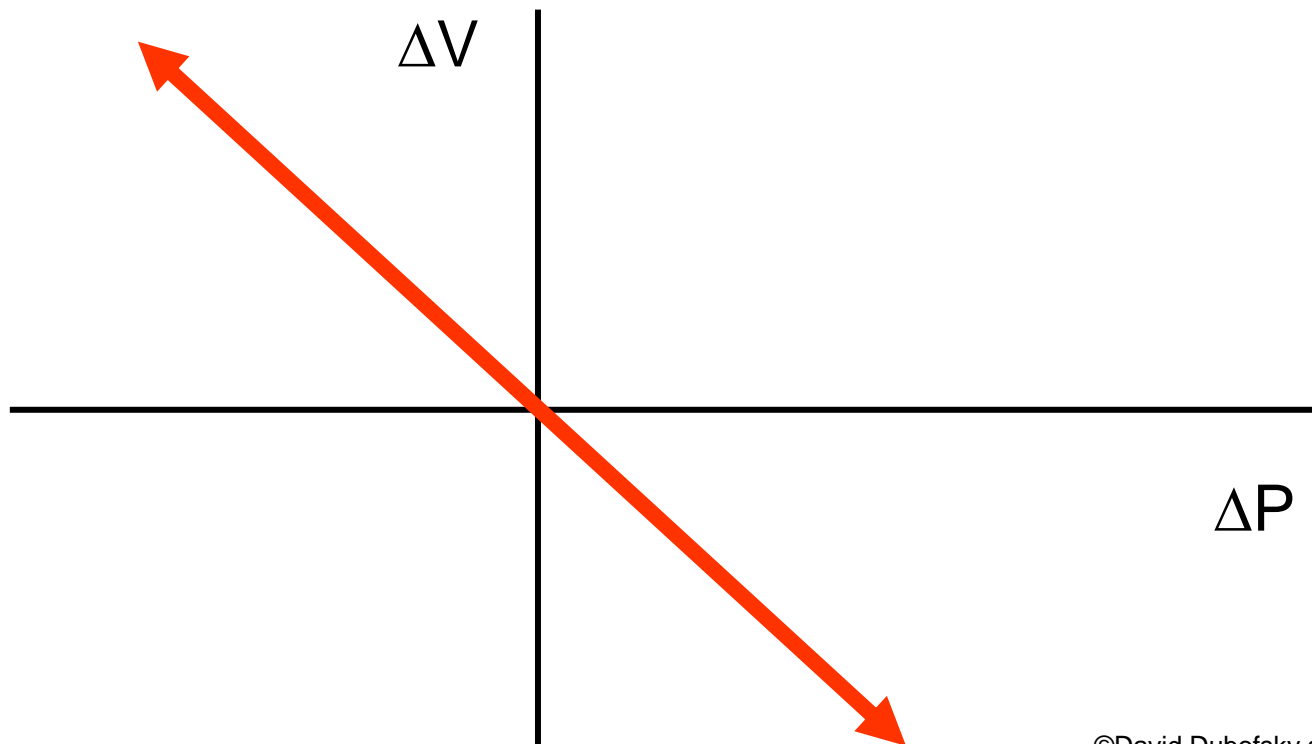


Buying the put = buying insurance

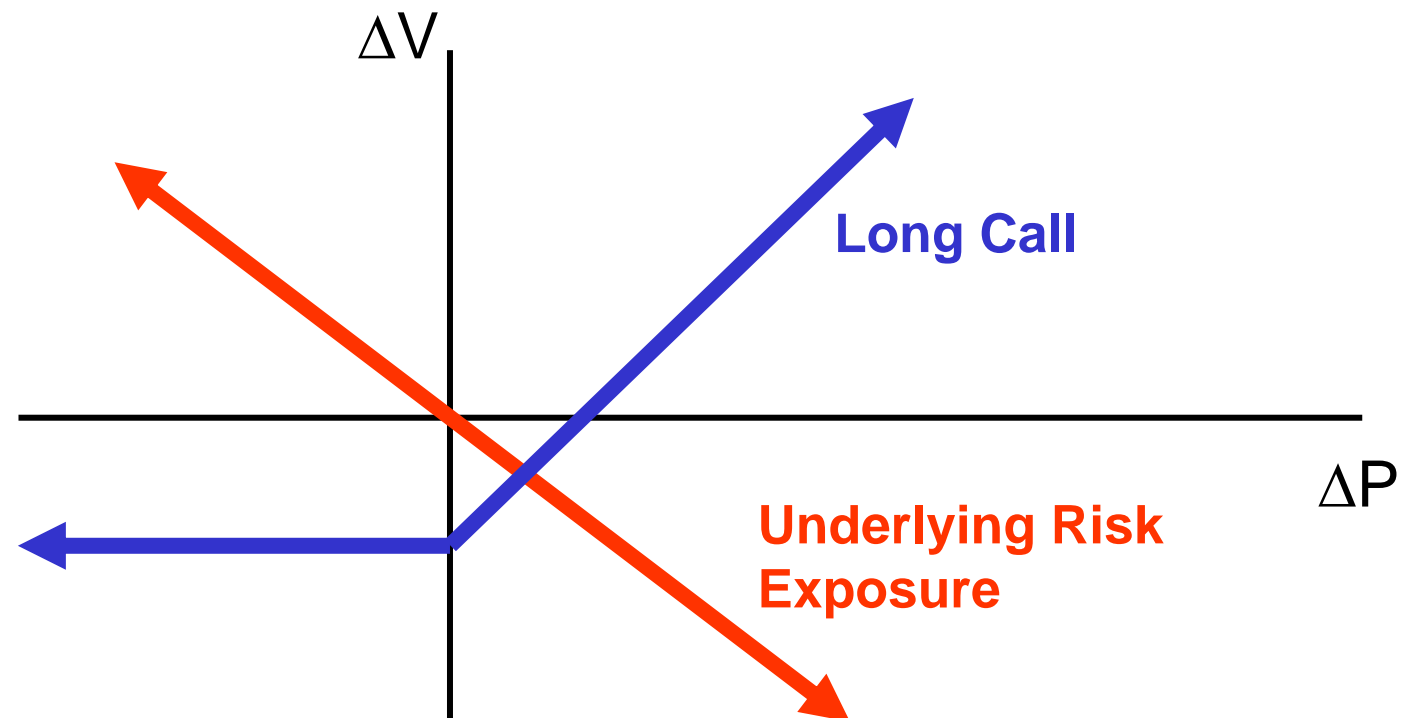


Using Options in Risk Management, II: Options Provide Insurance

Suppose a firm's value is 'exposed'
to the risk that some price will **increase**

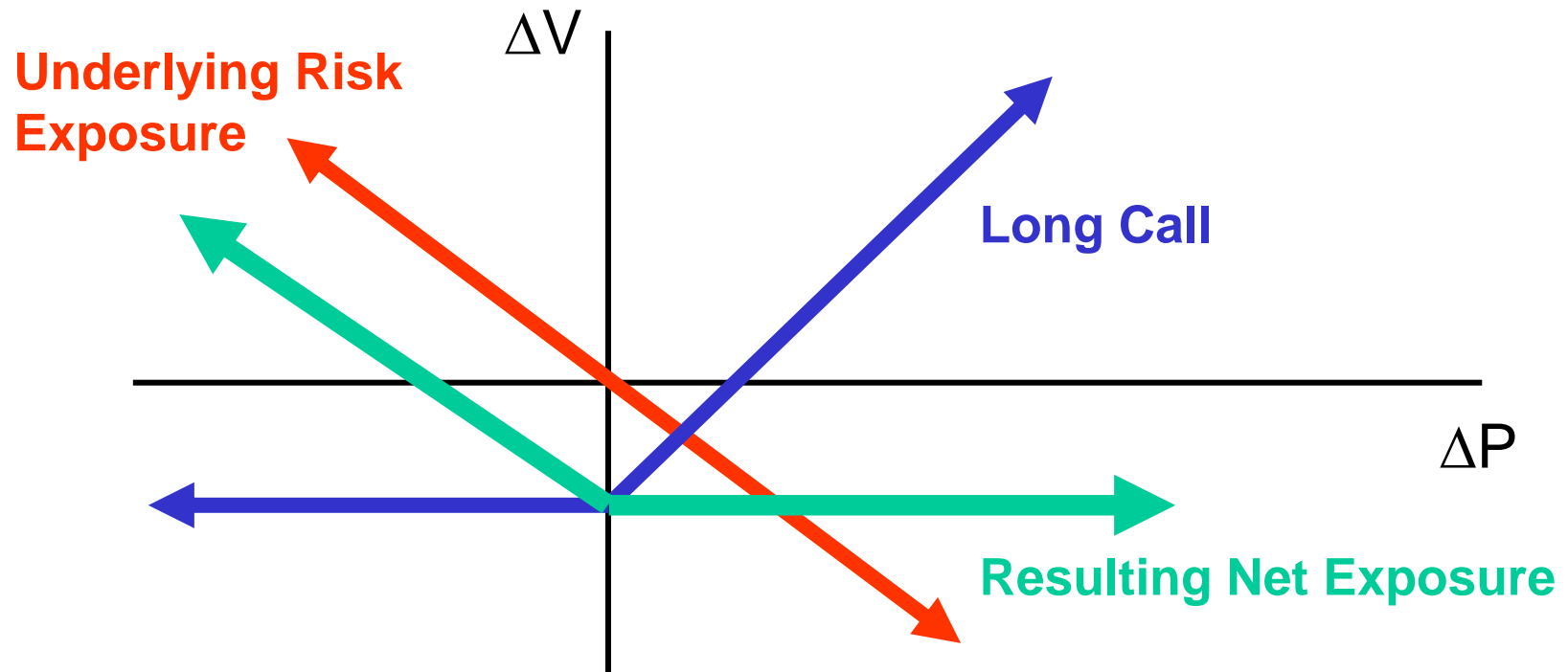


Buying a call option provides insurance against an increase in the price of an underlying security, an increase that would hurt the firm.



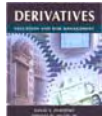
The resulting **Net Payoff** Profile of Adding a Long Call Option to a 'Short' Underlying Risk Position

Note that the firm is “hedged” if prices rise, but participates in the benefits should prices decline



Forwards and Futures and Financial Engineering

- You will see that all derivatives can be **replicated** by a combination of the underlying asset and borrowing/lending.
- For example: Long Forward = Borrow and Buy the Underlying Asset.
- Because Futures and Swaps are portfolios of Forwards, they also can be **replicated** using the underlying asset and borrowing/lending.

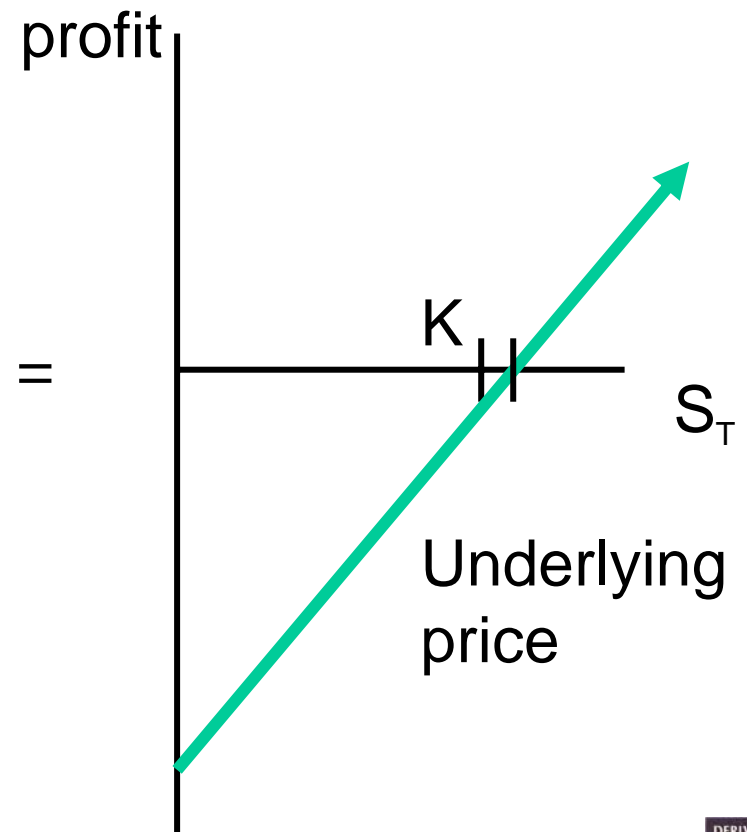
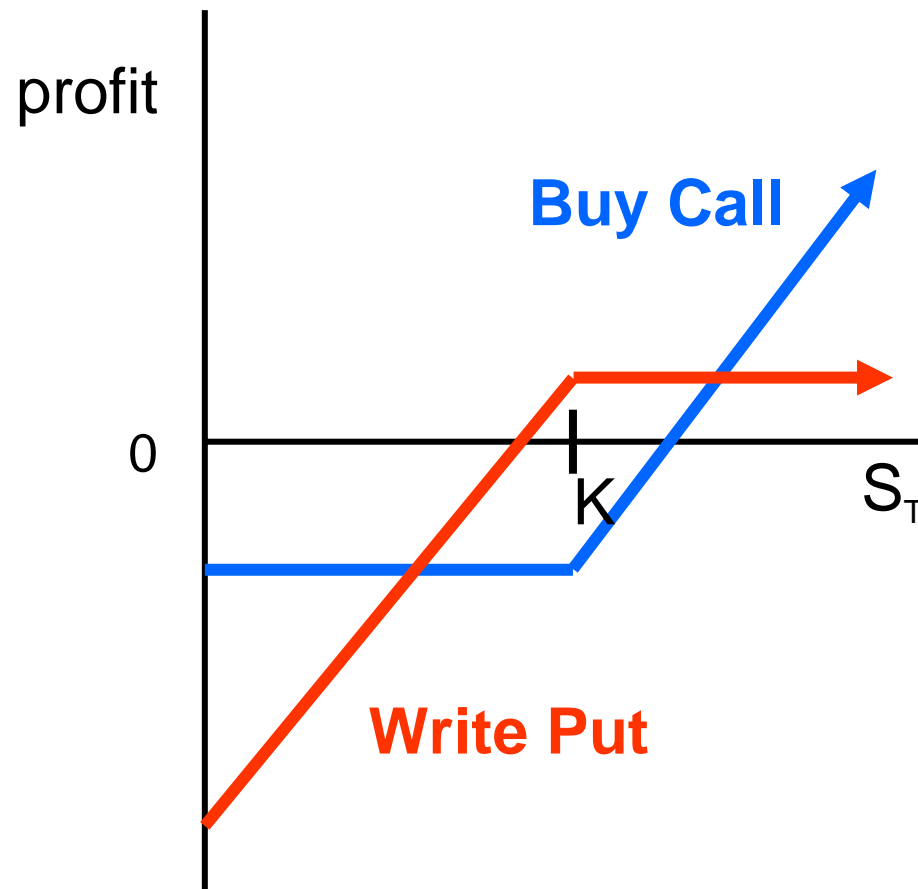


Options and Financial Engineering

- Options can be **replicated** by *dynamically* trading the underlying asset and borrowing/lending.
- Note that a position in a futures contract or a forward contract can be substituted for the position in the underlying asset.
 - Example I: Buy a Call and Write a Put = Long Underlying (or forward/futures).
 - Example II: Write a Call and Buy a Put = Short Underlying (or forward/futures).



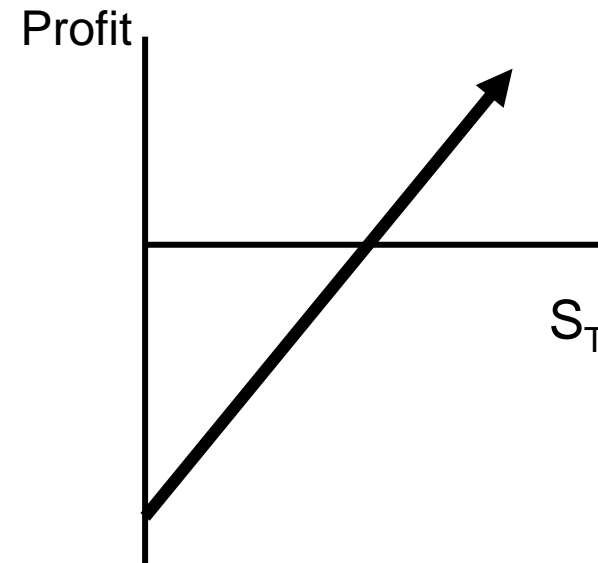
Buy a Call and Write a Put = Long Underlying



Showing that Buying a Call and Writing a Put Equals Long Underlying, Tabulated

- Let: $K=40$; $S=40$; $C=4$; $P=3$; $\longrightarrow CF_0 = -1$
- Then, at expiration (time T):

S_T	C_T	P_T	CF_T	$CF_0 + CF_T$
38	0	-2	-2	-3
39	0	-1	-1	-2
40	0	0	0	-1
41	1	0	1	0
42	2	0	2	1



Chapter 2

Risk and Risk Management

➤ Why Hedge? Prices are Volatile

- Interest Rates
 - interest rates are the price of credit, or the price of current consumption
- Foreign Exchange Rates
 - exchange rates are the prices for units of a foreign currency
- Stock Prices
- Commodity Prices
- Financial price risk is the effect that unexpected changes in the above have on profits and or value.



Prices are Volatile

- For about a 15 year history of interest rates, forex rates, stock indices, and commodities, see the chart rooms at <http://205.232.165.149/>.
- This is a link to Dr. Ed Yardeni's Economics Network.



Measuring Price Uncertainty

- Risk = uncertainty about the future. It is usually measured by the variance of a random variable. But this is not the only measure of uncertainty: range, Prob ($X < 0$), semivariance, etc., are also measures of dispersion.
- Estimation of variance (ex ante):

$$\begin{aligned}\text{Var}(\tilde{r}) &= E[r_i - E(\tilde{r})]^2 = \\ &= \sum_{i=1}^N [r_i - E(\tilde{r})]^2 \text{Prob}[\tilde{r} = r_i]\end{aligned}$$



Example of Calculating Variance

Outcome	Probability	Outcome times Probability	Squared Deviation from $E(r)$	Deviation times Probability
-0.08	0.2	-0.016	0.038809	0.007762
0.14	0.7	0.098	0.000529	0.000370
0.35	0.1	0.035	0.054289	0.005429

sum (= expected return): 0.117

sum (= variance): 0.013561

sq. root (= std devn): 0.116452

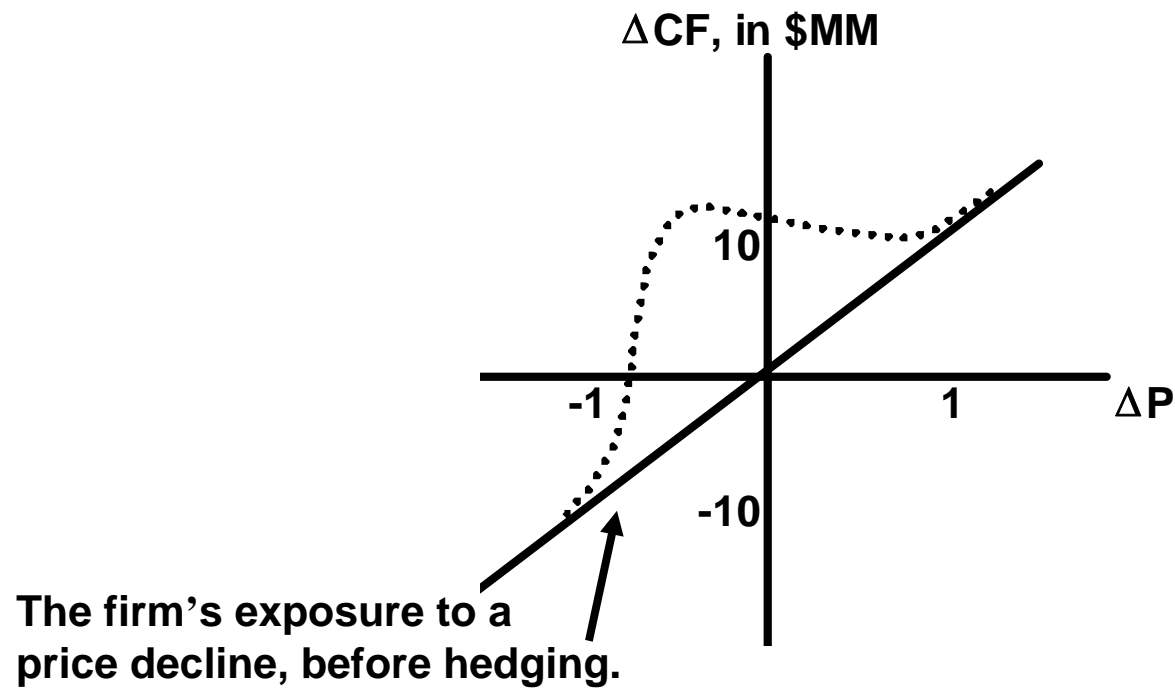


Measuring Price Uncertainty

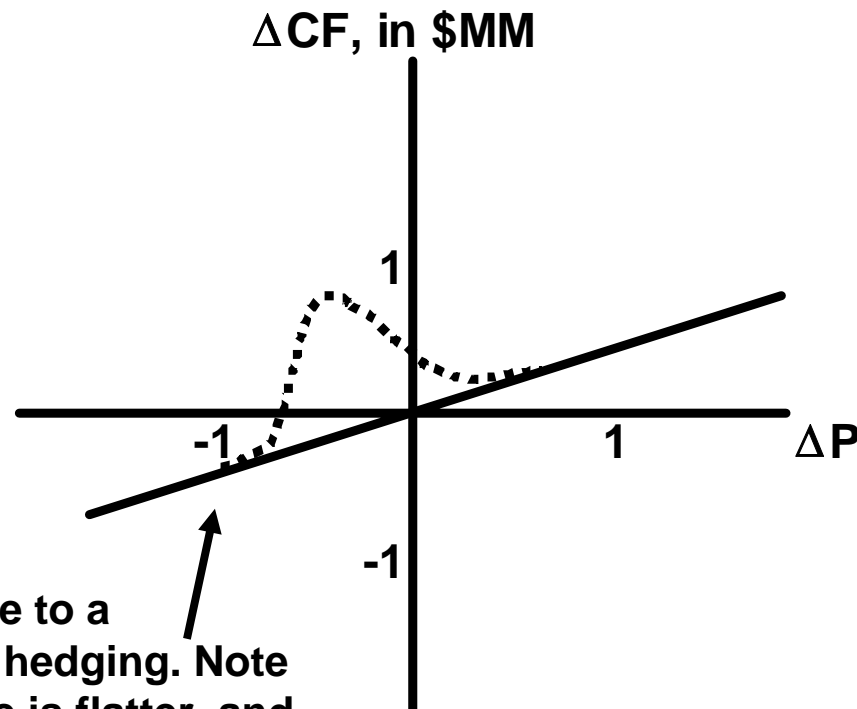
- Risk management reduces the variance of the random variable...it reduces uncertainty.
- Many professional risk managers try to reduce costs and/or increase profits by using derivatives only when they believe that doing so is to their advantage, i.e., “selective hedging”.
- Essentially, this is speculating.
- The alternative to selective hedging is “complete hedging”, or “continuous hedging”.



Hedging Can Lower the Variance (or Dispersion) of a Firm's Cash Flows



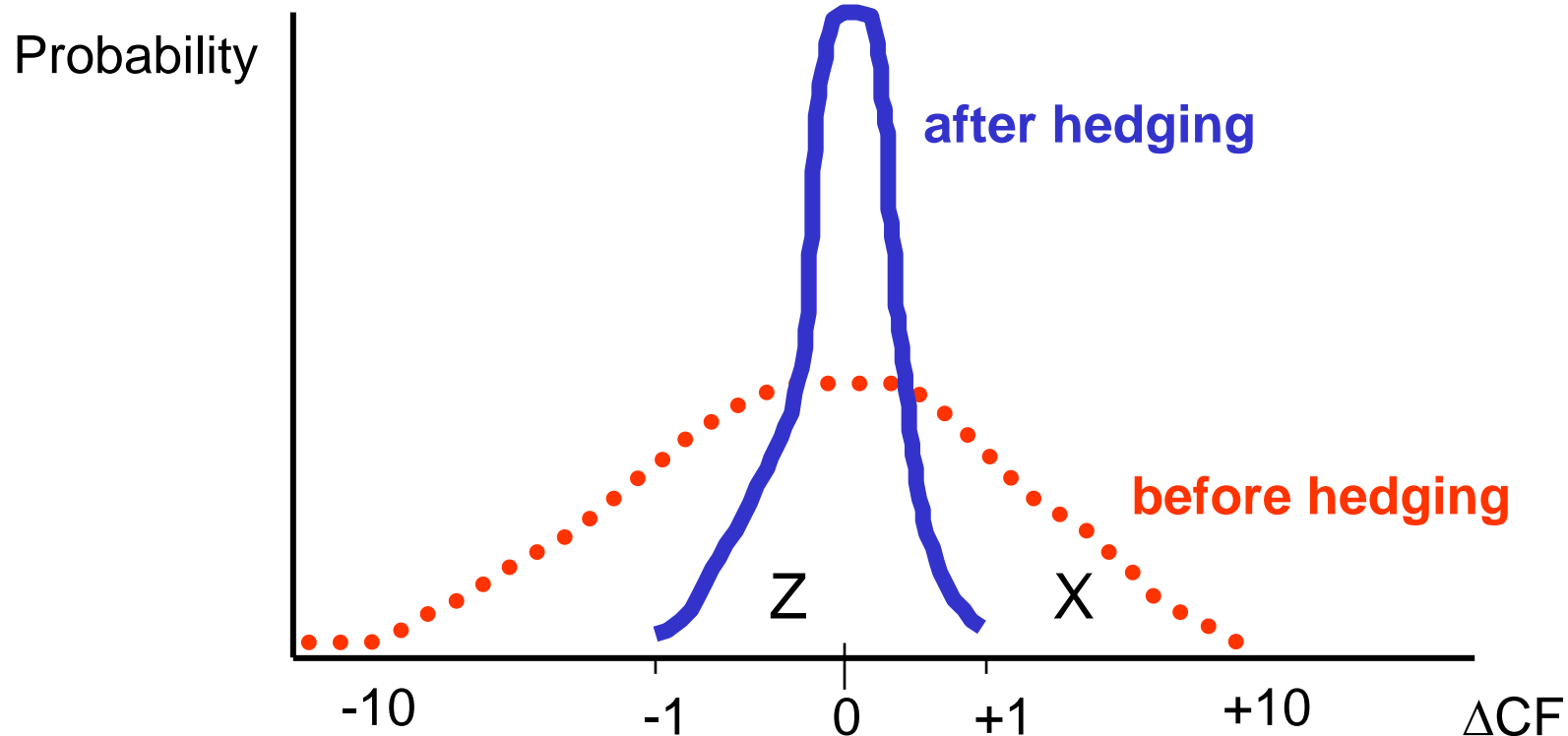
Hedging Can Lower the Variance (or Dispersion) of a Firm's Cash Flows



The firm's exposure to a price decline, after hedging. Note the slope of the line is flatter, and the outcome is less uncertain.



Hedging Can Lower the Variance, or Dispersion, of the Firm's Cash Flows




Note that “good” outcome X (pre-hedging) can become “bad” outcome Z (post-hedging)

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Risk Management

- Hedging and insurance does not **eliminate** risk. It only transfers it to those who are more willing and better able to confront risk and deal with it.
 - Transactional risk management versus overall economic risk management:
 - many transactions within the firm. Suppose unit A is exposed to the risk that $\$/\text{¥} \uparrow$ and unit B to the risk that $\$/\text{¥} \downarrow$?
-  One must identify the firm's **net** exposure.
- prices affect quantities bought or sold
 - competition
- Selective Hedging versus Complete Hedging



Determining the Extent of the Firm's Risk Exposure, I.

- Regressions of historical changes of cash flow on historical changes of prices, e.g.,

$$\Delta CF_t = \beta_0 + \beta_1 \Delta r_t + \beta_2 \Delta \text{£}_t + \beta_3 \Delta \text{oil}_t + \varepsilon_t$$

- Issues:
 - accounting numbers are subject to GAAP.
 - accounting numbers do not reflect opportunity costs.
 - regressions measure historical relationships.
 - did the firm engage in risk management in the past?



Determining the Extent of the Firm's Risk Exposure, II.

- Repeat the above regression model, except use stock returns as the dependent variable.
- Issues:
 - Investors may not realize the nature of the firm's risk exposures.
 - Equity returns are “noisy.”
 - Measures historical relationships; the future firm may differ from the past firm.



Determining the Extent of the Firm's Risk Exposure, III.

$$R_{it} = \beta_0 + \beta_1 R_{mt} + \beta_2 \left(\frac{\Delta r}{r} \right)_t + \beta_3 \left(\frac{\Delta fx}{fx} \right)_t + \beta_4 \left(\frac{\Delta Pc}{Pc} \right)_t + \varepsilon_{it}$$

This regression model estimates a time series of equity rates of return on:

- (1) the market's rate of return,
- (2) percent changes in interest rates,
- (3) percent changes in foreign exchange prices, and
- (4) percent changes in commodity prices.



Determining the Extent of the Firm's Risk Exposure, IV.

- Monte Carlo Simulation:
 - Develop a model of revenues and expenses that depend on financial prices.
 - Specify probability distributions of future financial prices.
 - Draw from the probability distributions and use model to determine net income.



Determining the Extent of the Firm's Risk Exposure, V.

- Repeat many times, and obtain a probability distribution of Net Income
- Issue: difficult to perform, and output is only as good as the model and the assumed probability distributions
- Benefit: Forward looking, and can perform sensitivity analysis on the assumptions.



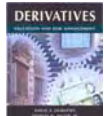
Should Firms Manage Risk?

- Quick answer from many observers: YES!
- Reasoning:
 - Managers are **risk averse**
 - Stockholders are **risk averse**
 - Stakeholders (i.e., employees, suppliers, customers) are **risk averse.**
- So, a reduction in risk should benefit everyone.
- Plus, a decrease in volatility implies a lower cost of capital, and therefore, an increase in firm value.
- Risk aversion: a willingness to pay a premium to reduce risk exposure.



An Important Finance Theory:

- Under perfect markets, financial decisions such as risk management will not affect firm value. After all, if stockholders can manage risk themselves, why pay managers to do it?
- Should the risk management group be allowed to hedge “selectively”? Or, should the goal of the firm be to minimize the firm’s risk exposure?
- ***MOST*** firms “selectively hedge.”
- A 1998 survey revealed that 3 in 5 large non-financial U.S. firms used derivatives.



“Perfect Market” Assumptions

- No taxes.
- No transactions costs.
- No costs of financial distress.
- All market participants are price takers.
- All market participants have **equal access** to all information.



Therefore, if Hedging, (i.e. Risk Shifting) is to have Value:

- It has to be because of these “market imperfections.”
- That is, “real-world” stuff.
- Examples of how and when hedging adds value:

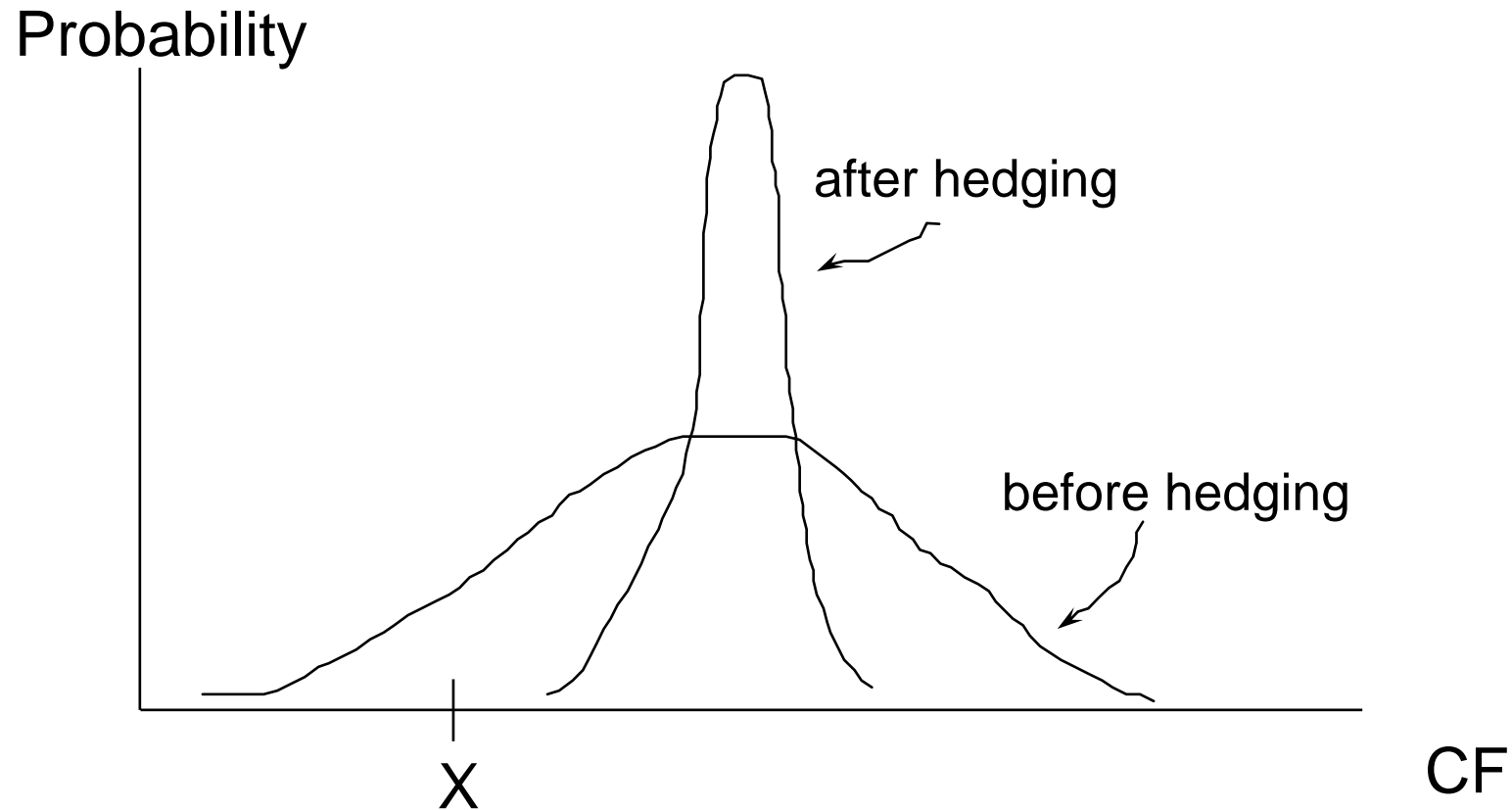


Why Hedge? I. Hedging Reduces the Costs of Financial Distress

- Example: A cattle rancher can increase debt capacity because cash flows are known.
- Costs of Financial Distress Include:
 - Customers fear the firm won't be able to stand by its warranties, hence, lower sales.
 - Suppliers are reluctant to extend trade credit.
 - Employees demand higher wages.
 - Management time diverted.
 - Investment opportunities may be 'overlooked'



Risk management can reduce the costs of financial distress



X = level of CF where costs of financial distress begin to appear



Why Hedge? II. Hedging makes it more likely that future attractive investments will be made by the firm.

- Example: a financially distressed firm has \$3 to invest in either project A or project B.
- Debt principal due next year is \$6.

<u>Probability</u>	<u>Payoff One Year Hence:</u>	
	<u>Project A</u>	<u>Project B</u>
0.30	\$5	\$0
0.39	\$5	\$0
0.30	\$5	\$0
0.01	\$5	\$10

Bondholders
Prefer Project A.

Shareholders Will
Pick Project B.



This is, of course, an example of an Agency Problem

- Bondholders recognize this behavior before they buy bonds, and demand to be compensated for this risk.
- By hedging, financial distress becomes less likely, and therefore the added interest costs of debt, and the possible failure to accept positive NPV projects, are averted.
- Lowering the volatility of cash flows increases the likelihood that the firm will have the cash needed to make new investments.
- If so, the firm will not be forced to resort to external financing in order to invest.



Why Hedge? III. Hedging may allow a firm to use its tax deductible items in the year they are generated.

	<u>Unhedged Scenario</u>		<u>Hedged Scenario</u>	
	<u>Growth</u>	<u>Recession</u>	<u>Growth</u>	<u>Recession</u>
Probability	50%	50%	50%	50%
Gross profit	300	0	150	150
- depreciation expense	(80)	(80)	(80)	(80)
- interest expense	(20)	(20)	(20)	(20)
Taxable Income	200	(100)	50	50
- taxes (30%)	(60)	0	(15)	(15)
Net Income	140	(100)	35	35
Net Cash Flow	220	(20)	115	115
Expected Net Income	20		35	
Expected Net Cash Flow	100		115	



What this Example Shows Us, I.

- Hedging reduces the likelihood that negative taxable income is earned. I.e., hedging reduces the probability that a firm will be forced to carry tax losses forward. When tax losses are carried forward, their time value is lost.
- But even beyond the lost time value, hedging also increases average net income and cash flow because of the tax code.
- Note that this example assumes that tax loss carry-backs cannot be used. If the firm was supplied with a \$30 tax rebate in the event that taxable income is -\$100, then expected net income and expected cash flow would be the same, hedged or unhedged.
- Note that using tax loss carry-forwards is not as desirable as being able to use tax deductible items immediately, due to the time value of money.



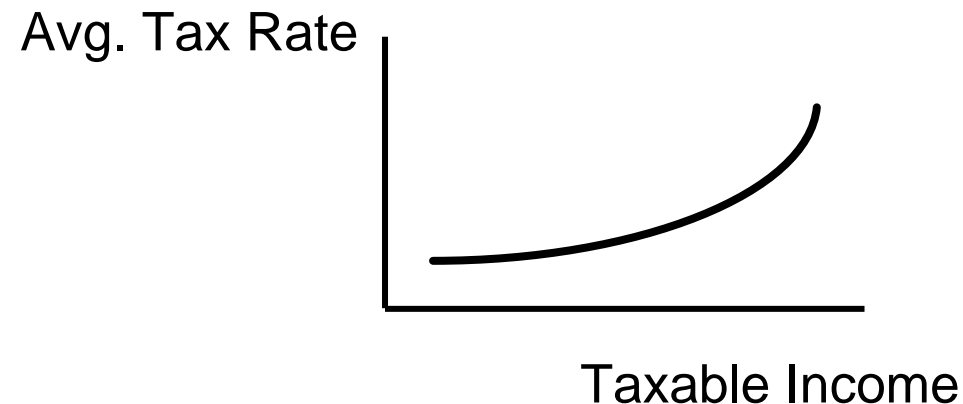
What this Example Shows Us, II.

- This example shows that average net income and average cash flow are greater when the firm is hedged.
- If the hedged firm's average cash flow is discounted at the same or lower discount rate, then we can conclude that hedging increases the value of the firm.
- If the market applies the same or higher price earnings multiple to the hedged firm's average earnings per share, then again we must conclude that hedging increases the value of the firm's common stock.



A Progressive Tax Rate Schedule, i.e., marginal rates rise as Taxable Income rises, produces a Convex Tax Schedule.

Taxable Income	Tax Rate
< \$100,000	0
\$100,000 to \$200,000	5% on amt over \$100,000
\$200,000 to \$300,000	\$5,000 + 15% on amt. over \$200,000
\$300,000 to \$400,000	\$20,000 + 30% on amt. over \$300,000
> \$400,000	\$50,000 + 50% on amt. over \$400,000



Why Hedge? IV. Hedging Reduces Average Taxes Paid When the Tax Schedule is Convex.

Unhedged

Prob.	Taxable Income	Taxes
0.50	\$150,000	\$2,500
0.50	\$450,000	\$75,000

Expected taxes: \$38,750

Hedged

Prob.	Taxable Income	Taxes
1.00	\$300,000	<u>\$20,000</u>



Why Hedge? More Reasons.

- It is less costly for a firm to hedge than for a stockholder.
- Firms may have better **market information** than individuals.
 - Cocoa Market: You versus Hershey's
 - Coffee Market: You versus Starbuck's
 - Certainly, firms know about specific transactions they are going to enter.
- Non-systematic risk should be hedged when owners are not well diversified.
- Risk averse managers will prefer to hedge, regardless of whether they should or shouldn't. But, compensation packages can be tailored to align the interests of managers and stockholders.



Other Issues in Risk Management

- Direction of exposure versus extent of exposure.
- Will a 200 basis point rise in interest rates merely reduce profits, or lead to bankruptcy?
- An excellent site dealing with financial price risk management is: **<http://www.garp.com>**



Legal Issues

- In one UK case, it was illegal for a counter-party to engage in the transaction --- and the courts ruled that it was therefore OK to walk away from the obligation.
- The implications of another case is that corporations have a duty to hedge.
- Should fiduciaries have a duty to hedge?
- At one time, there was risk that OTC derivatives could be declared illegal.
- Industry regulation (e.g., banking), and market regulation (SEC, CFTC).



Accounting Issues

- SEC (62 Fed. Reg 6,044 (Feb. 10, 1997) requires that public companies provide investors with information about their derivatives activities.
- FASB 133, which went into effect in the year 2000.
 - All derivatives must be recognized as assets or liabilities and measured at **fair value** for financial accounting purposes.
 - Changes in the fair value of derivatives that are **not** used for hedging must be recognized as a gain or loss in the period of change.
 - Different accounting for hedging vs. trading.



Some Tax Issues

- General tax treatment differs for the four basic types of derivatives.
- Should trades be marked to market at year's end? (OTC forwards and options are deferred until the contract's settlement date; exchange traded contracts are marked to market.)
- Ordinary income vs. capital gains/losses.
- There are special rules governing taxation if the derivatives trade is part of a hedge. (But, what about a complicated, or a strategic, hedge?)



Some Extra Slides on this Material

- Note: In some chapters, we try to include some extra slides in an effort to allow for a deeper treatment of the material in the chapter.
- If you have created some slides that you would like to share with the community of educators that use our book, please send them to us!



Recent Derivative Debacles:

- Rogue Trader at Fault
 - Baring's Bank
 - Sumitomo Bank
 - Daiwa Bank
 - Kidder Peabody
 - Midland Bank
 - Orange County
 - Shell
- Treasury Department at Fault
 - Proctor and Gamble
 - Gibson Greeting
 - Metallgesellschaft
- Long Term Capital Management



Risk Management

- Despite these highly publicized cases, derivative markets are here to stay.
- Today, 3 in 5 large non-financial U.S. firms use derivative securities.
- An important lesson: separate the funding of trading from trading decisions.
- Two overarching principals:
 - Do not take on risk if your goal is to shed risk.
 - If you do not understand the derivative product, do not trade.



A Very Hard Question: What is the firm's overall net exposure to price risks?

- Unit A: exposed to dollar price of yen **increasing**.
- Unit B: exposed to dollar price of yen **decreasing**.
- So, it's the **net exposure** that must be identified.



Another Very Hard Question: Should Firms Manage Risk?

- Quick answer from many observers: YES!
- Reasoning:
 - Managers are **risk averse**
 - Stockholders are **risk averse**
 - Stakeholders (i.e., employees, suppliers, customers) are **risk averse.**
- So, a reduction in risk should benefit everyone.
- Plus, a decrease in volatility implies a lower cost of capital, and therefore, an increase in firm value.



What is Risk Aversion?

- Definition: Willing to pay a premium to reduce uncertainty.
- Risk averse managers want to shrink the range of uncertainty.
- Uncertainty measured by:
 - Volatility
 - Variance
 - Standard Deviation



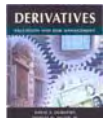
A Big, Important Decision

- Should the risk management group be allowed to hedge “selectively”?
- Or, should the goal of the firm be to minimize the firm’s risk exposure?
- ***MOST*** firms “selectively hedge.”
- Today, 3 in 5 large non-financial U.S. firms use derivative securities.



Does “Selective Hedging” Equal Speculation?

- Yes.
- But, ‘speculation’ is not *evil*.
- Speculation, of any kind, is simply **taking** a price risk.
- Keeping a risk, versus shifting it, is speculating.
- Taking a risk you don’t currently have is speculating.



Chapter 3

Introduction to Forward Contracts

- Both futures and forwards specify a trade between two counter-parties:
 - There is a commitment **to deliver** an asset (this is the **seller**, or the **short**), at a specified **forward price**.
 - There is a commitment **to take delivery** of an asset (this is the **buyer**, or the **long**), at a specified **forward price**.
 - At delivery, cash is exchanged for the asset.
- Many times, futures contracts and forward contracts are substitutes. However, at other times, the relative costs, liquidity, and convenience of using one market versus the other will differ.

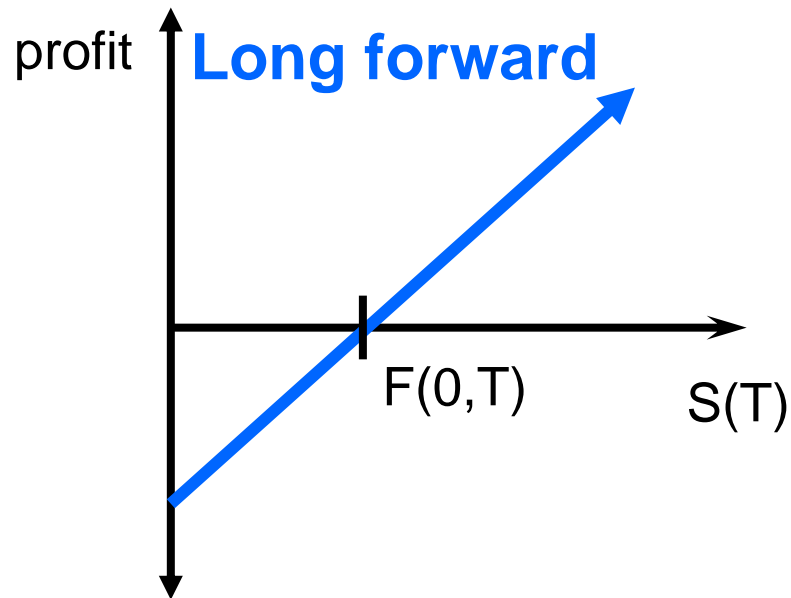


Futures and Forwards: A Comparison Table

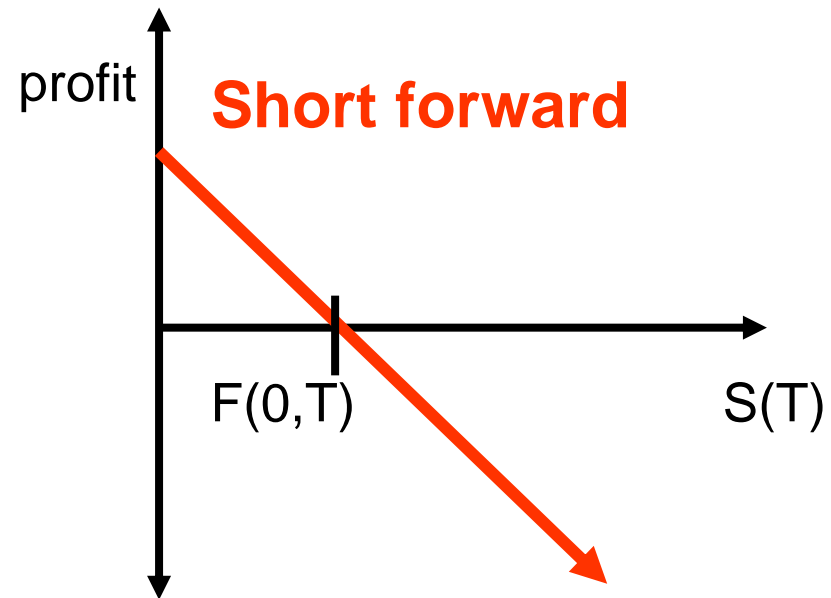
	<u>Futures</u>	<u>Forwards</u>
Default Risk:	Borne by Clearinghouse	Borne by Counter-Parties
What to Trade:	Standardized	Negotiable
The Forward/Futures Price	Agreed on at Time of Trade Then, Marked-to-Market	Agreed on at Time of Trade. Payment at Contract Termination
Where to Trade:	Standardized	Negotiable
When to Trade:	Standardized	Negotiable
Liquidity Risk:	Clearinghouse Makes it Easy to Exit Commitment	Cannot Exit as Easily: Must Make an Entire New Contract
How Much to Trade:	Standardized	Negotiable
What Type to Trade:	Standardized	Negotiable
Margin	Required	Collateral is negotiable
Typical Holding Pd.	Offset prior to delivery	Delivery takes place



Forward Contracts: Payoff Profiles



The long profits if the spot price at delivery, $S(T)$, exceeds the original forward price, $F(0,T)$.



The short profits if the price at delivery, $S(T)$, is below the original forward price, $F(0,T)$.



Profits for Forward Contracts

- If $S(T) > F(0,T)$, the long profits by $S(T) - F(0,T)$ per unit, and the short loses this amount.
- If $S(T) < F(0,T)$, the short profits by $F(0,T) - S(T)$ per unit, and the long loses this amount.
- Example: You sell ¥20 million forward at a forward price of \$0.0090/ ¥. At expiration, the spot price is \$0.0083/ ¥.
 - Did you profit or did you lose?
 - How much?



Default Risk for Forwards, I.

- If the forward price is “fair” at initiation:
 - The contract is valueless.
 - There is no immediate default risk.
- As time goes by, the forward price (for delivery on the same date as the original contract subsequently) can change:
 - Existing forward contracts acquire value: They become an asset for one party and a liability for the other.
 - Default risk appears. (Q: Which party faces default risk?)



Default Risk for Forwards, II.

- At any time, only **one** counter-party has the *incentive* to default.
- It is the counter-party for whom the forward contract has become a liability.
- The amount exposed to default risk at time t is:

$$PV\{ |F(0,T) - F(t,T)| \}, \quad 0 < t < T$$



Default Risk for Forwards, III.

- Example: On April 4th, you buy 100,000 bbl of oil forward at a forward price of \$27/bbl. The delivery date is July 4th. On May 4th, the forward price for delivery on July 4th is \$23/bbl.
 - Who has the incentive to default?
 - If the interest rate on May 4th for 2-month debt instruments is 5%, what is the dollar amount exposed to default risk?



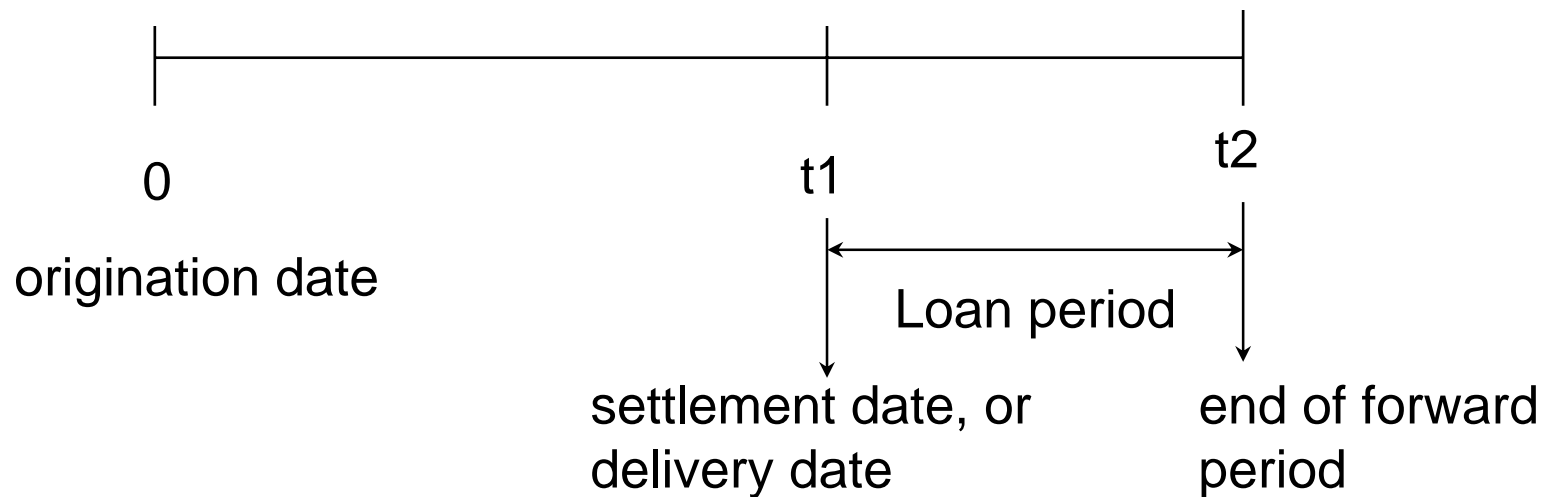
Forward Rate Agreements (FRAs)

- A FRA is a forward contract *on an interest rate* (not on a bond, or a loan).
- The **buyer** of a FRA **profits** from an **increase** in interest rates. The **seller** of a FRA **profits** from a **decline** in rates.
- The buyer *effectively* has agreed to borrow an amount of money in the future at the stated forward (contract) rate. The seller has effectively locked in a lending rate.
- FRA's are cash settled.



Forward Rate Agreements (FRAs)

- Only the difference in interest rates is paid. The principal is not exchanged.
- FRAs are cash settled.



Forward Rate Agreements (FRAs)

- If the spot rate at delivery (“settlement rate”, $r(t_1, t_2)$) exceeds the forward rate agreed to in the FRA (“contract rate” = $fr(0, t_1, t_2)$), the FRA buyer profits.
- The amount paid (at time t_1) is the present value of the difference between the settlement rate and the contract rate times the notional principal times the fraction of the year of the forward period.
- A FRA’s value is initially zero, when the contract rate is the theoretical forward rate. Subsequently, forward rates will change, so the FRA will have positive value for one party (and equally negative value for the other).



Forward Rate Agreements (FRAs)

- Let $r(t_1, t_2)$ = settlement rate (spot rate at delivery)
- Let $fr(0, t_1, t_2)$ = contract rate = original forward rate
- Let D = days in contract period = $t_2 - t_1$
- Let P = notional principal
- Let $B = 360$, (or 365 sometimes)
- Cash settlement payment equals:

$$\left| \frac{P[r(t_1, t_2) - fr(0, t_1, t_2)](D/B)}{1 + [r(t_1, t_2)](D/B)} \right|$$



FRA “Jargon”

- A t_1 X t_2 FRA: The start date, or delivery date, is t_1 months hence. The end of the forward period is t_2 months hence. The loan period is t_1 - t_2 months long.
- E.g., a 9 X 12 FRA has a contract period beginning nine months hence, ending 12 months hence.



An Example of an FRA

- A firm sells a 5X8 FRA, with a NP of \$300MM, and a contract rate of 5.8% (3-mo. forward LIBOR).
- On the settlement date (five months hence), 3 mo. spot LIBOR is 5.1%.
- There are 91 days in the contract period ($8-5=3$ months), and a year is defined to be 360 days.
- Five months hence, the firm receives:



FRA Example (continued)

- \$524,077.11, which is calculated as:

$$\frac{(300,000,000)(0.058-0.051)(91/360)}{1+[(0.051)(91/360)]}$$



Test Your Understanding.

- A firm sells a 5X8 FRA, with a NP of \$300MM, and a contract rate of 5.8% (3-mo. forward LIBOR).
- Suppose that one month after the origination of the FRA, 4X7 FRAs are priced at 5.5% and 5X8 FRAs are priced at 5.6%.
- Also assume that that time, the spot four month interest rate is 4.9%.
- What is the original FRA worth?



FRA and Interest Rate Swaps

- A FRA is essentially equivalent to a one-period plain vanilla interest rate swap:

FRA

Buyer

Seller

Swap

pay fixed ($fr(0,t_1,t_2)$), and
receive floating ($r(t_1,t_2)$)

receive fixed and pay floating



Foreign Exchange

- It is important here to understand the difference between the dollar price of an fx, and the fx price of a dollar.
- It is also important to understand the difference between the € price of a £, and the £ price of a €.
- There are spot exchange rates and forward exchange rates (see the WSJ).
- See <http://www.bloomberg.com/markets/fxc.html> for spot rates.



Profits and Losses for Forward FX Contracts

- Define $F(0,T)$ as the forward exchange rate at origination, for N units of FX to be delivered at time T
- $S(T)$ = the spot exchange rate at delivery.
- $F(0,T)$ and $S(T)$ are in $\$/fx$.
- If $S(T) > F(0,T)$, the long profits by: $N [S(T) - F(0,T)]$.
- If $S(T) < F(0,T)$, the short profits by: $N [F(0,T) - S(T)]$.



Profits and Losses for Forward FX Contracts: An Example

- Note the WSJ reprint on the next slide. Observe the spot rate, and the forward exchange rate for the yen for delivery 3 months hence.
- If the spot rate remains unchanged from today until the delivery date, what is the profit/loss for a party who goes long a forward contract on ¥10,000,000?



Exchange Rates

The foreign exchange mid-range rates below apply to trading among banks in amounts of \$1 million and more, as quoted at 4 p.m. Eastern time by Reuters and other sources. Retail transactions provide fewer units of foreign currency per dollar.

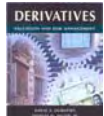
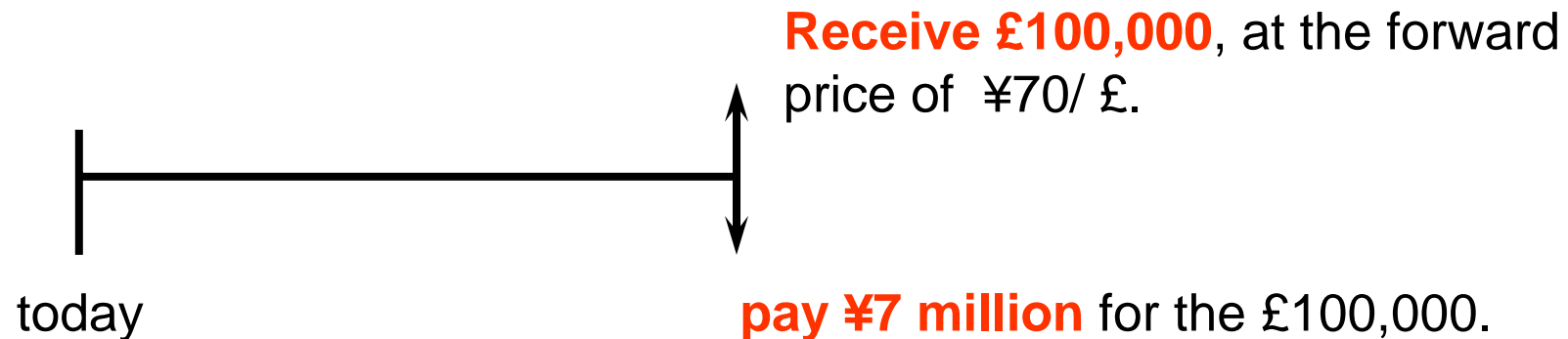
Country	U.S. \$ EQUIVALENT		CURRENCY PER U.S. \$	
	Wed.	Tue.	Wed.	Tue.
Argentina (Peso)-y	.2747	.2759	3.6400	3.6250
Australia (Dollar)	.5450	.5485	1.8347	1.8233
Bahrain (Dinar)	2.6525	2.6525	.3770	.3770
Brazil (Real)	.3213	.3220	3.1125	3.1060
Britain (Pound)	1.5643	1.5636	.6393	.6395
1-month forward	1.5614	1.5607	.6405	.6407
3-months forward	1.5560	1.5553	.6427	.6430
6-months forward	1.5478	1.5472	.6461	.6463
Canada (Dollar)	.6380	.6440	1.5674	1.5527
1-month forward	.6374	.6435	1.5688	1.5541
3-months forward	.6362	.6421	1.5718	1.5575
6-months forward	.6342	.6398	1.5768	1.5630
Chile (Peso)	.001397	.001396	715.95	716.25
China (Renminbi)	.1208	.1208	8.2771	8.2770
Colombia (Peso)	.0003714	.0003722	2692.40	2686.50
Czech. Rep. (Koruna)				
Commercial rate	.03263	.03275	30.645	30.531
Denmark (Krone)	.1335	.1342	7.4925	7.4500
Ecuador (US Dollar)	1.0000	1.0000	1.0000	1.0000
Hong Kong (Dollar)	.1282	.1282	7.8000	7.8000
Hungary (Forint)	.004044	.004058	247.25	246.43
India (Rupee)	.02065	.02064	48.430	48.450
Indonesia (Rupiah)	.0001127	.0001131	8870	8843
Israel (Shekel)	.2116	.2129	4.7270	4.6965
Japan (Yen)	.008475	.008542	118.00	117.07
1-month forward	.008488	.008556	117.82	116.88
3-months forward	.008512	.008580	117.48	116.55
6-months forward	.008546	.008615	117.01	116.08
Jordan (Dinar)	1.4184	1.4184	.7050	.7050
Kuwait (Dinar)	3.3190	3.3212	.3013	.3011



A Forward FX Contract is a Pair of Zero Coupon Bonds, I.

One is an asset, and the other is a liability denominated in a different currency.

Consider the forward contract to buy £100,000 for the forward price of ¥70/ £.



A Forward FX Contract is a Pair of Zero Coupon Bonds, II.

- This firm has an asset: it owns a zero coupon bond that promises to pay off £100,000 at maturity.
- It also has a liability: It has effectively issued a zero coupon bond, and must pay investors ¥7MM at maturity.
- Because there is no cash flow at origination, the value of each zero coupon bond must be equal (denominated in any currency).
- That is, the PV of £100,000 must equal the PV of ¥7MM, given the spot exchange rate and interest rates in Britain and Japan.



Chapter 4

Using Forwards to Manage Risk

- Trading forward and futures contracts (or other derivatives) with the objective of reducing price risk is called ***hedging***.
- Not all risks faced by a business can be hedged — consider ***quantity risk***.



Hedging Fundamentals

- Hedging with futures/forwards *typically* involves taking a position in a futures market that is *opposite* the position already held in a cash market. ←
- **A Short (or selling) Hedge:** Occurs when a firm holds a long cash position and then **sells** futures/forward contracts for protection against downward price exposure in the cash market.
- **A Long (or buying) Hedge:** Occurs when a firm holds a short cash position and then **buys** futures/forward contracts for protection against upward price exposure in the cash market. Also known as an ***anticipatory hedge***.
- **A Cross Hedge:** Occurs when the asset underlying the futures/forward contract differs from the product in the cash position.
- Firms can hold long and short hedges simultaneously (but for different price risks).



An Income Statement View

- An elementary income statement is:

Revenues (= output price times units sold)

-Costs (= input prices times units of inputs purchased)

Profits

If output prices decline (all else equal), or if input prices rise (all else equal), then the firm's profits will decline.



A Balance Sheet View

- An elementary balance sheet is:

Assets

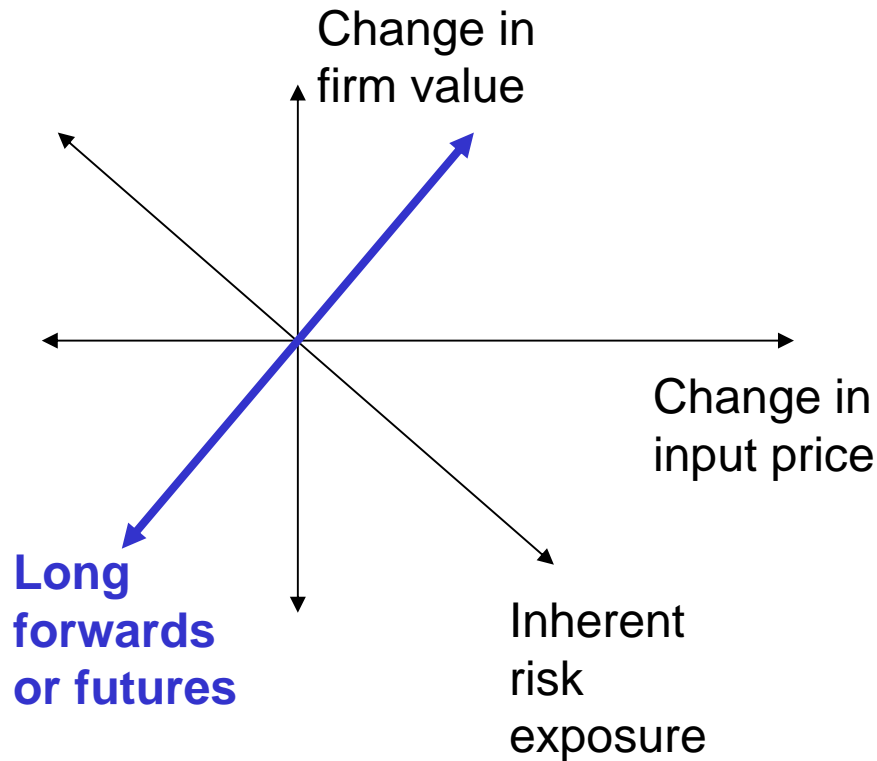
Liabilities

Owners' Equity (stock value)

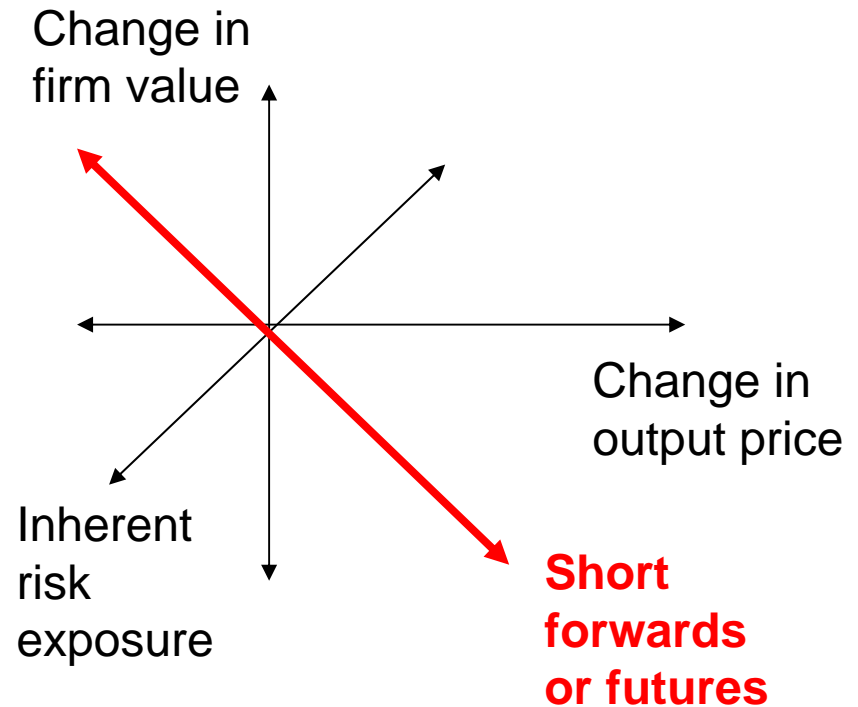
- Any price change that decreases the value of a firm's assets, relative to its liabilities, will hurt the stockholders.
- Any price change that increases the value of a firm's liabilities, relative to its assets, will hurt the stockholders.



A Diagrammatic View



Firm faces risk that input prices will rise. **Long** hedge is appropriate.

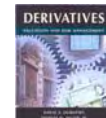
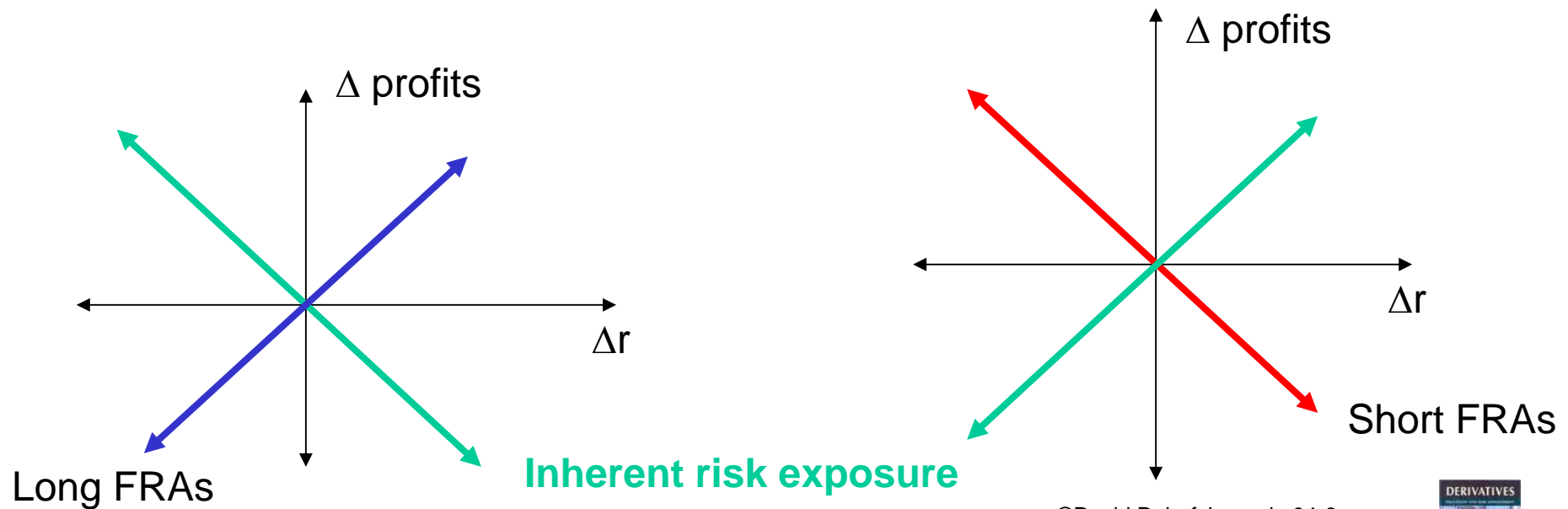


Firm faces risk that output price will decline. **Short** hedge is appropriate.



Hedging Interest Rate Risk with FRAs

- Buy FRAs to hedge against rising interest rates.
- Sell FRAs to hedge against falling interest rates



How a FRA Can Lock in a Lending Rate

- A money market fund knows it will have \$50 million to lend three months hence.
- It plans on lending the money for one year. $r(0,3) = 4\%$, $r(0,12) = 5\%$, $r(0,15) = 5.2\%$. $fr(0,3,15) = 5.45\%$.
- The fund fears that interest rates will _____ (rise or fall?). Therefore, it will _____ (buy or sell?) a ___ X ___ FRA.



12-month LIBOR, 3 months hence	Interest Income paid 15 months hence	Realized Profit/loss on FRA, 3 months hence
5%	$(0.05)(\$50\text{MM}) = \2.5MM	\$214,285.71
6%	$(0.06)(\$50\text{MM}) = \3MM	(\$261,904.76)
Future Value of profit/loss on FRA	Total effective interest income	Effective lending rate
\$225,000	\$2,725,000	0.0545
(\$275,000)	\$2,725,000	0.0545

Example: If 12-month LIBOR, 3 months hence, is 5%, the realized profit of \$214,285.71 on the FRA is computed as:

$$\frac{50,000,000(0.0545 - 0.0500)(365/365)}{[1+0.05(365/365)]}$$



Managing Currency Price Risk, I.

- Consider a U.S.-based firm that wants to maximize dollar denominated profits. Its revenues are in dollars, but its expenses are in yen. The income statement is:

$$\begin{array}{r} \$\text{Revenues} \\ - \text{¥Costs} \\ \hline \text{Profits} \end{array}$$

- Does the firm fear that the yen will rise or fall in value relative to the dollar ($\$/\text{¥} \uparrow$ or $\$/\text{¥} \downarrow$)?
- To hedge, will the firm want to buy or sell yen forward?



Managing Currency Price Risk, II.

- Consider a U.S.-based firm that wants to maximize dollar denominated stockholder wealth. Its assets are in dollars, but its liabilities are in yen. The balance sheet is:

\$Assets	¥Liabilities
	Owners' Equity (stock value)

- Does the firm fear that the yen will rise or fall in value ($\$/¥ \uparrow$ or $\$/¥ \downarrow$)?
- To hedge, will it want to buy or sell yen forward?



Managing Currency Price Risk, III.

- Consider a British-based firm that wants to maximize £-denominated profits. Its revenues are in euros, but its expenses are in pounds. The income statement is:

€Revenues
-£Costs
Profits

- Does the firm fear that the euro will rise or fall in value (£/ €↑ or £/ €↓)?
- To hedge, will it want to buy or sell euros forward?



Managing Currency Price Risk, IV.

- Consider a German-based firm that wants to maximize €-denominated stockholder wealth. Its assets are in yen, but its liabilities are in euros. The balance sheet is:

¥Assets	€Liabilities
	Owners' Equity (stock value)

- Does the firm fear that the yen will rise or fall in value ($€/¥ \uparrow$ or $€/¥ \downarrow$)?
- To hedge, will it want to buy or sell yen forward?



Buy forward to hedge against a price increase, a balance sheet view

- A British firm's only fx exposure is that it owes (accounts payable) euros to a German supplier.
- A firm has a substantial investment in long term Treasuries, but no interest sensitive liabilities.
- A firm owns a bauxite mine. If aluminum prices rise by only a small amount, demand for it will plummet (AL demand is price _____ (elastic or inelastic?)), causing a decline in the value of the mine.



Sell forward to hedge against a price decrease, a balance sheet view

- A Japanese firm owns some real estate in the U.S., otherwise, it has no foreign assets or liabilities.
- A firm's liabilities consist of fixed rate debt.
- A firm's most important asset is a patent it has on transforming water into oil. At the current price of crude oil, the process is economical. At lower oil prices, it is not.



Test Your Comprehension, I:

- A German firm borrowed £20 million at a floating interest rate. The 3-month Sterling rate at time t determines the interest payment at time $t+1$. Payments are made quarterly. The yield on 3-month Sterling is 6% today.
-
- What interest rate risk does it face? How can it use a FRA to manage this risk?
- What exchange rate risk does it face? How can it use a forward exchange contract to manage its risk exposure?



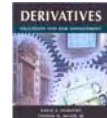
Test Your Comprehension, II:

- A Swiss fixed income mutual fund has invested SFR50 million in long term German bonds having a coupon rate of 6%. The current exchange rate is SFR0.84/€.
- What interest rate risk does it face? How can it use a FRA to manage this risk exposure?
- What exchange rate risk does it face? How can it use a forward exchange contract to manage this risk exposure?

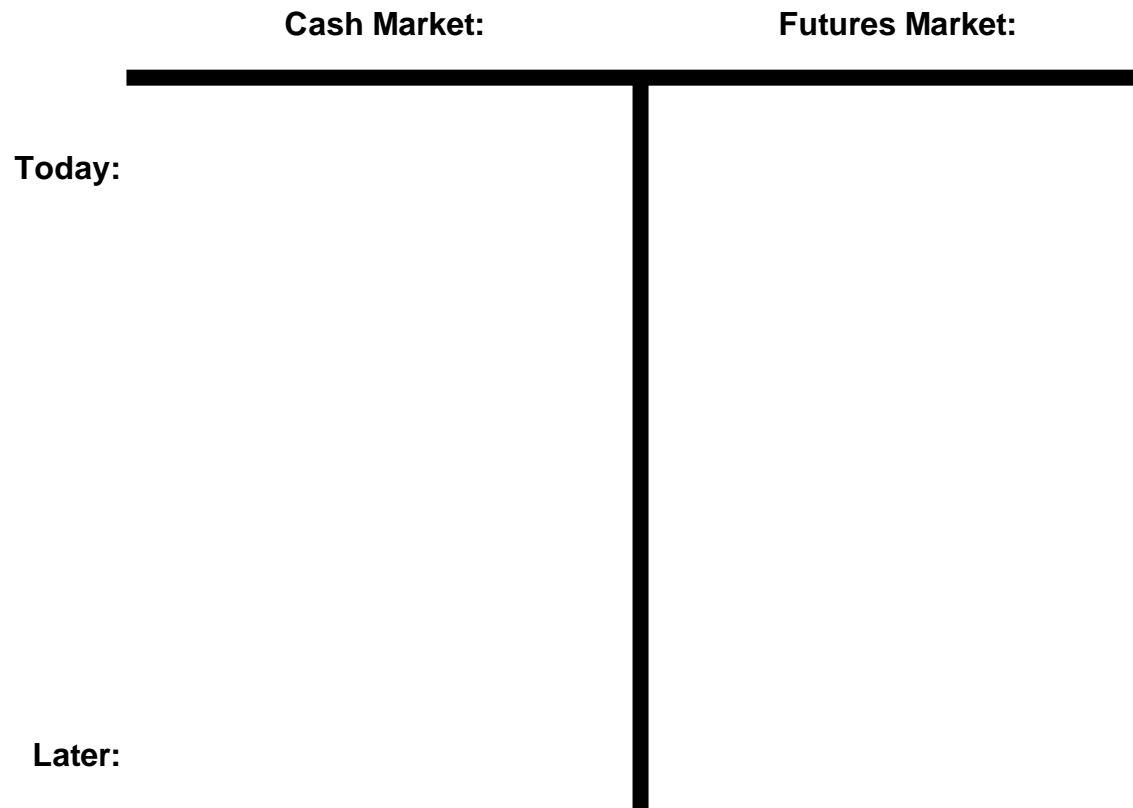


Some Extra Slides on this Material

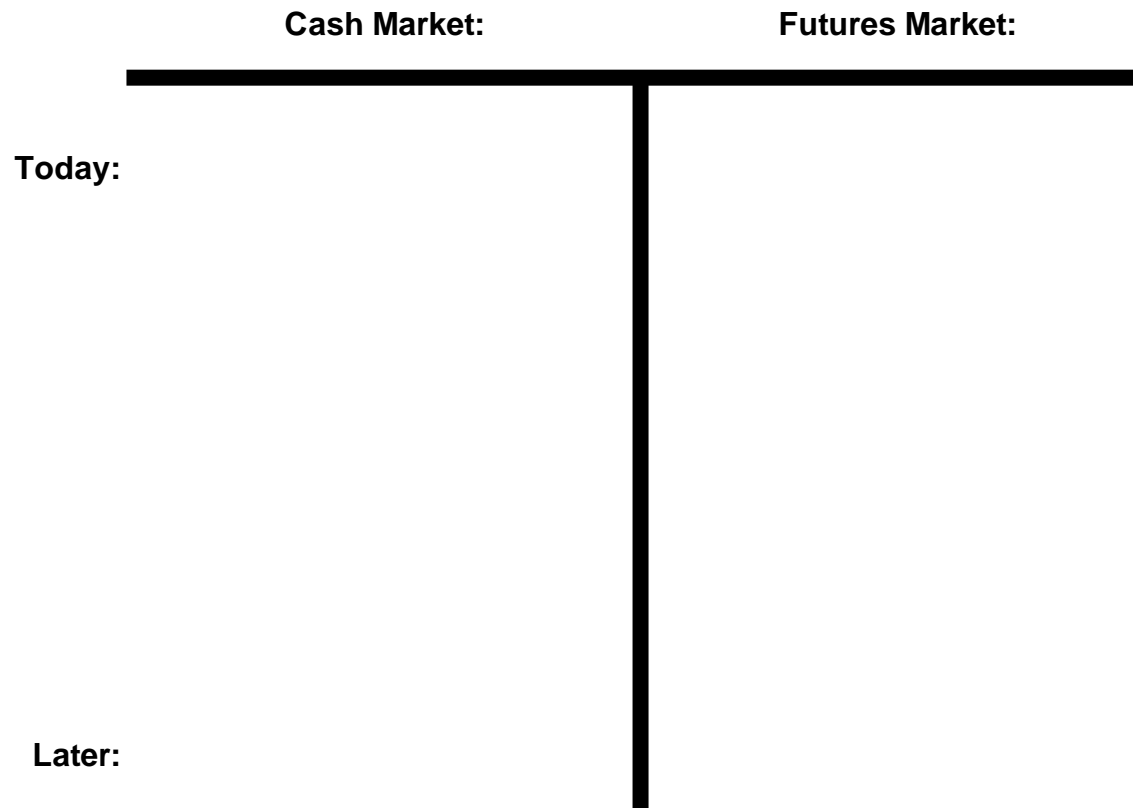
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- If you have created some slides that you would like to share with the community of educators that use our book, please send them to us!



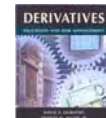
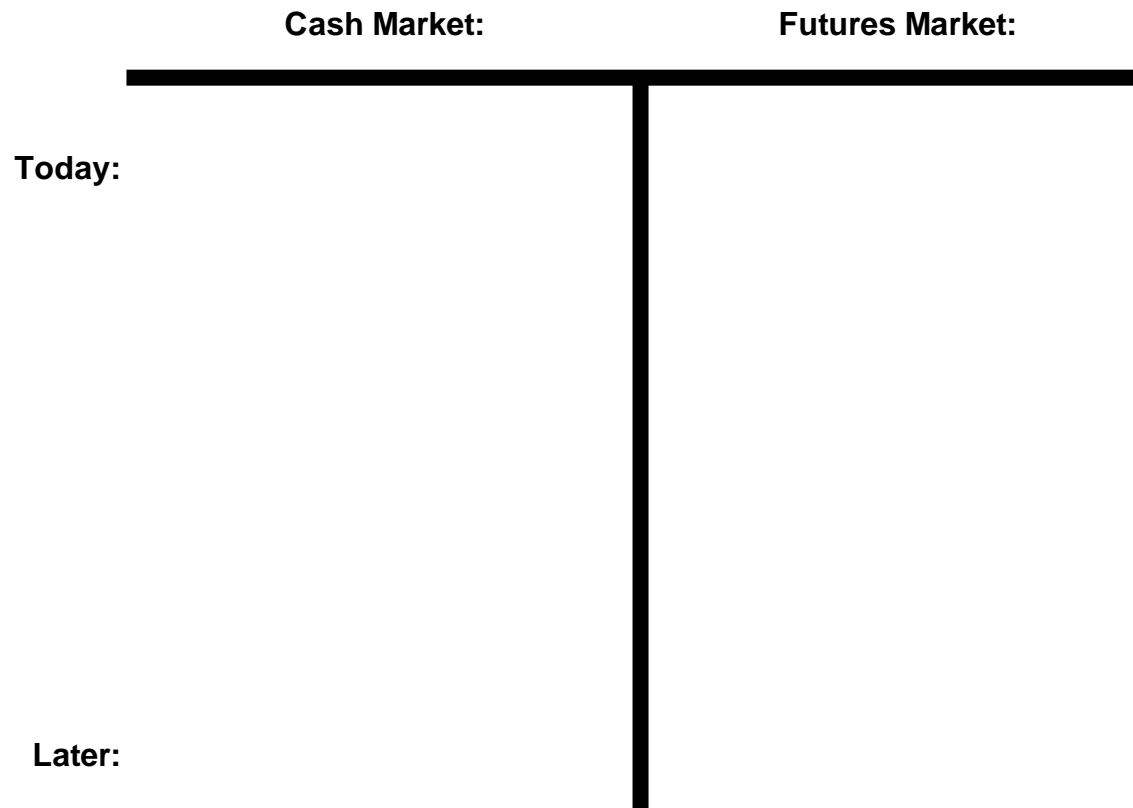
The T-Account. *Fill in the position the firm will take in the cash market later. This is the position the firm should take in the futures market today.*



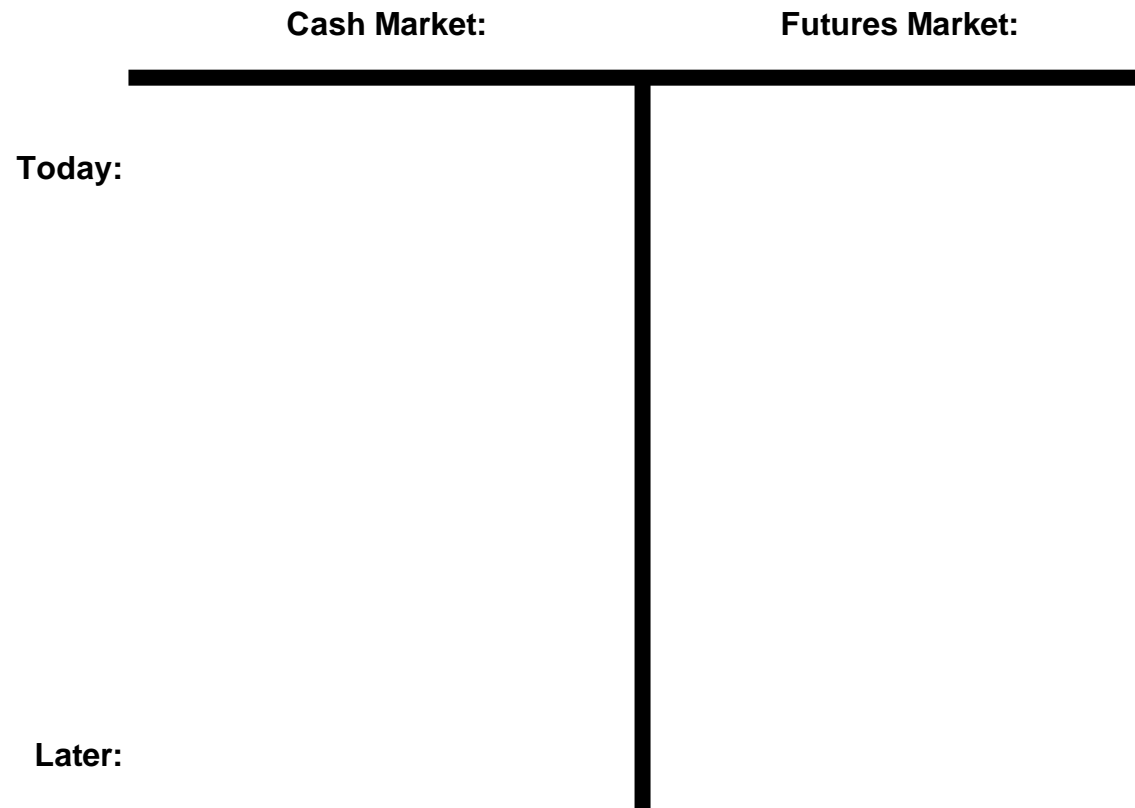
Practice: The Hedge for an Oil Refinery in the Output Market (i.e., Gasoline and Heating Oil)



Practice: The Hedge for an Oil Refinery in the Input Market (i.e., Crude Oil)



Practice: The Hedge for a Pension Fund Manager Looking to “Lock-in” Current Bond Yields



A Numerical Example.

A Gold Mining Hedger: Base Case

		Oz. Mined:	
Revenue:	\$37,000,000	100,000	Avg. Selling Price: 370
Variable Costs:	\$20,000,000	100,000	Cost: 200
Fixed Costs:	<u>\$10,000,000</u>		
Pre-Tax Profit:	\$7,000,000		
Taxes (40%)	<u>\$2,800,000</u>		
Profit	<u><u>\$4,200,000</u></u>		
Return on Equity:	16.8%	Equity:	25,000,000



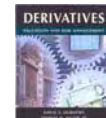
A Gold Mine Hedger: Good Case

Revenue:	\$40,000,000	Oz. Mined:	100,000	Avg. Selling Price:	400
Variable Costs:	\$20,000,000		100,000	Cost:	200
Fixed Costs:	<u>\$10,000,000</u>				
Pre-Tax Profit:	\$10,000,000				
Taxes (40%)	<u>\$4,000,000</u>				
Profit	<u><u>\$6,000,000</u></u>				
Return on Equity:	24.0%			Equity:	25,000,000

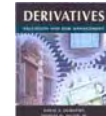
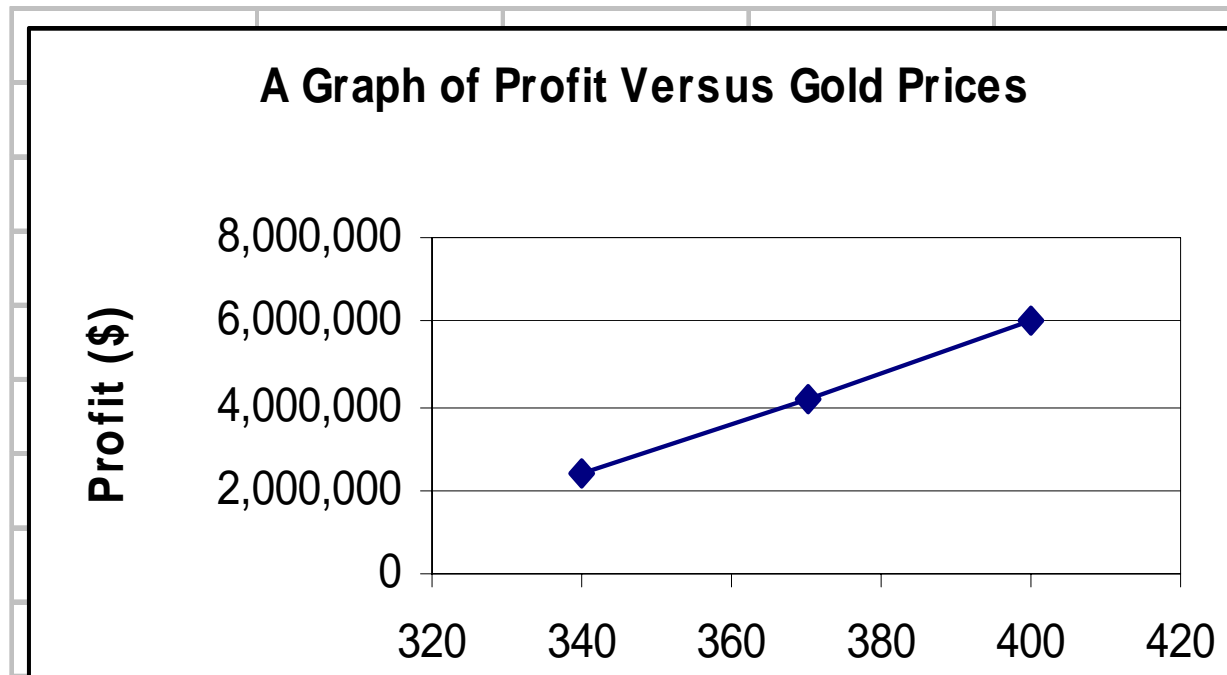


A Gold Mine Hedger: Bad Case

Revenue:	\$34,000,000	Oz. Mined:	100,000	Avg. Selling Price:	340
Variable Costs:	\$20,000,000		100,000	Cost:	200
Fixed Costs:	<u>\$10,000,000</u>				
Pre-Tax Profit:	\$4,000,000				
Taxes (40%)	<u>\$1,600,000</u>				
Profit	<u><u>\$2,400,000</u></u>				
Return on Equity:	9.6%			Equity:	25,000,000

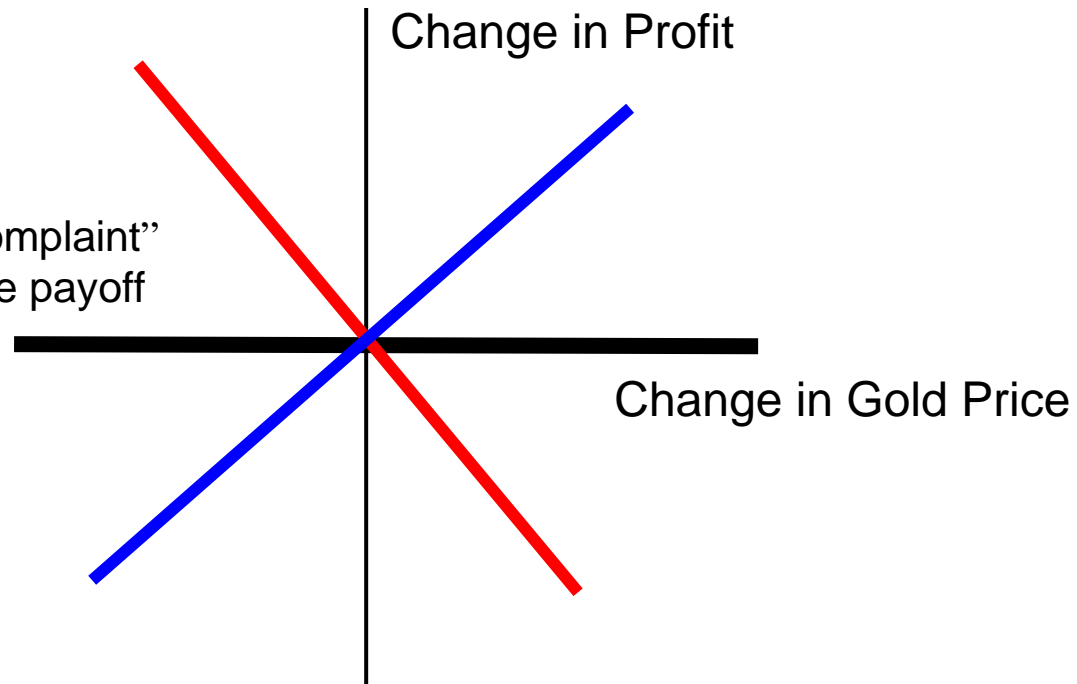


Profit Versus Gold Prices



Generalized Profit Profile with Hedging (the Cross)

Q: What's the "Complaint" about flattening the payoff profile?



Chapter 5

Determining Forward and Futures Prices

- In a well functioning market, the forward price of carry-type commodities (stocks & stock indexes, debt securities, currencies, & gold) must preclude the possibility of arbitrage.
- That is, at any time 't':

$$F = S + CC - CR$$

where:

F = **THEORETICAL** Futures Price at time t

S = Spot Price at time t

CC = Carrying Costs from time t to time T

CR = Carry Returns from time t to time T



Forward Pricing: Cash and Carry Arbitrage

- Ignore, for now, the carry return (CR), as well as carrying costs such as storage and insurance costs
- What if $F > S + CC = S(1+h(0,T)) = S + h(0,T)S$?

- Today

borrow	+S
buy the good	-S
sell the good forward	<u> </u>
$CF_0 =$	0

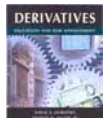
- At delivery

repay loan w/ interest	-S(1+h(0,T))
sell at forward price	<u>+F</u>
$CF_T =$	$F - S(1+h(0,T)) > 0$ ARBITRAGE PROFIT!



Forward Pricing: Cash and Carry Arbitrage

- $h(0,T)$ = the unannualized interest rate = $rT/365$
 - T = days until delivery
 - r = the annual interest rate
- If the set of cash and carry trades entails no cash flow at time 0, there must be no cash flow at time T (delivery).
- Arbitrage: A set of trades requiring no initial investment, no risk, and a positive return.
- If $F - S(1+h(0,T)) > 0$, then by borrowing to buy the spot good, and selling it forward, you can arbitrage.
- Conclusion: F cannot be greater than $S(1+h(0,T))$; $F \leq S(1+h(0,t))$



The Perfect Market Assumptions for the Cost of Carry Relationship

- There are no commissions.
- There are no bid-ask spreads.
- There are no taxes.
- Market participants have no influence over price (price takers).
- Market participants want to maximize wealth.
- There are no impediments to short-selling.
- Short-sellers have full use of the short-sale proceeds.
- There is an unlimited ability to borrow or lend money.
- All borrowing and lending is done at the same interest rate.
- There is no default risk associated with buying or selling in either the forward or spot market.
- Commodities can be stored indefinitely without any change in the characteristics of the commodity (such as its quality).



Forward Pricing: Reverse Cash and Carry Arbitrage

- What if $F < S(1+h(0,T))$?

- Today

sell the good	+S
lend the proceeds	-S
buy the good forward	_____
	$CF_0 = 0$

- At delivery

the loan repays you w/ interest	+S(1+h(0,T))
buy at forward price	<u>-F</u>
	$CF_T = -F + S(1+h(0,T)) > 0$ ARBITRAGE!

Conclusion: F cannot be less than $S(1+h(0,T))$



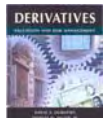
Problems with Reverse Cash and Carry Arbitrage

- The arbitrageur must either sell the good out of inventory, or sell it short.
- Selling it short requires that you find someone who is willing to lend it to you, and that you get full use of the proceeds.
- Selling it out of inventory means that you won't have use of the good (convenience yield); important for non-carry commodities (everything except financials and gold).



Conclusions about Forward Pricing

- Assume no transaction costs, no carry return and no costs of storing, (like insurance).
- For non-carry commodities: $F \leq S(1+h(0,T))$. Prices cannot permit cash and carry arbitrage.
- For gold and financials (carry commodities):
 $F = S(1+h(0,T)) = S + Sh(0,T) = S + \text{interest}$. Prices cannot permit either cash and carry arbitrage or reverse cash and carry arbitrage.



An Example

- Spot gold sells for \$403/oz. The six month interest rate is 4.5%; the one year interest rate is 5% (both are annual rates).

- Assume no transaction costs and no storage, etc. costs.

- For there to be no arbitrage, the forward price of gold for delivery six months hence must be:

$$403(1.0225) = 412.0675.$$

- The forward price of gold for delivery one year hence must be:

$$403(1.05) = 423.15.$$



FinCad Result:

aaCDF		
spot price per unit of underlying com	403	
rate - annual compounding	0.045	
value (settlement) date	23-Sep-97	
expiry date	22-Mar-98	
accrual method	2	actual/ 360
storage cost	0	
convenience value	0	
statistic	2	fair value
fair value	412.0675	
aaAccrual_days		
effective date	23-Sep-97	
terminating date	22-Mar-98	
accrual method	2	actual/ 360
number of business days from an eff	180	



Cash and Carry Arbitrage: An Example

- What if the actual forward price of gold for delivery 6 months hence is 413? [that is, $FP_0 > S(1+h(0,T))$]

- Today (at zero cash flow):

borrow: \$403

buy 1 oz. of gold for \$403

sell gold forward at $FP_0 = \$413$

- Six months hence:

repay loan: $403(1.0225) = -\$412.0675$

sell gold: +\$413

arbitrage profit = \$0.9325/oz.



Reverse Cash and Carry Arbitrage: An Example

- What if the forward price of gold for delivery one year hence is 422? [that is, $FP_0 < S(1+h(0,T))$]
- Today (at zero cash flow):
 - sell gold for: \$403
 - lend: \$403
 - buy gold forward at $FP_0 = \$422$
- One year hence
 - get repaid: $403(1.05) = +423.15$
 - buy gold: -422.00
 - arbitrage profit = \$1.15/oz.



Cash and Carry Arbitrage With Storage and Insurance Costs (CC_0)

- Today

borrow

$$+S+CC_0$$

buy the good

$$-S$$

pay storage and insurance costs

$$-CC_0$$

sell the good forward

$$CF_0 = \frac{\quad}{0}$$

- At delivery

repay loan w/ interest

$$-(S+CC_0)(1+h(0,T))$$

sell at forward price

$$F$$

$$CF_T = \frac{F}{F-(S+CC_0)(1+h(0,T))}$$

Hence, F must equal $(S+CC_0)(1+h(0,T))$, or there will be an arbitrage opportunity.



Cash and Carry Arbitrage: Good that Provides a Carry Return

- What must be the forward price of a share of stock, for delivery six months hence, if $S_0 = 40$, $r = 6\%$, and the stock will pay a quarterly dividend of \$0.30/share one month and four months hence?



Cash and Carry Arbitrage: Good that Provides a Carry Return, I.

- Today ($CF_0 = 0$)

borrow \$40	+S	(+40)
buy the stock for \$40	-S	(-40)
sell the good forward		

- One month hence ($CF = 0$)

receive the \$0.30 dividend	+D ₁	(+0.30)
lend the \$0.30 dividend	-D ₁	(-0.30)

- Four months hence ($CF = 0$)

receive the \$0.30 dividend	+D ₂	(+0.30)
lend the \$0.30 dividend	-D ₂	(-0.30)



Cash and Carry Arbitrage: Good that Provides a Carry Return, II.

- At delivery

repay loan w/ interest (\$41.20) ¹	-S(1+h)
receive divs w/ interest (\$0.6105) ²	+FV(divs)
sell good at forward price	+F
$CF_T =$	F-S(1+h)+FV(divs)

- Conclude that $F = S(1+h) - FV(\text{divs})$
- $F = 41.20 - 0.6105 = \$40.5895$
- Sometimes spot-futures parity is written as $F = Se^{(r-d)t}$. This expression assumes continuous carry costs and carry returns.



Notes on the Forward Price of a Stock Example, Previous Slide

- ¹ $(40)(1.03) = 41.20$.
- ² The 1st dividend earns 5 months of interest: $[(6\%)(5/12) = 2.5\%]$.
The 2nd dividend earns 2 months of interest: $[(6\%)(2/12) = 1\%]$. Thus,
the future value of dividends is: $\$0.30(1.025) + 0.30(1.01) = \0.6105 .
Mathematically:

$$FV(\text{Divs}) = \sum_{t=0}^T \text{Div}_t (1 + h(t, T))$$



FinCad Result:

aaEqty_fwd		
value (settlement) date	23-Sep-97	
expiry date	23-Mar-98	
accrual method	4	30/ 360 <input type="button" value="v"/>
rate - annual compounding	0.06	
cash price of the underlying equity in	40	
statistic	2	fair value of forward or futures <input type="button" value="v"/>
dividend payment table	t_14	
t_14		
dividend payment table		
all the dividend dates from the value	dividend amount	
	23-Oct-97	0.3
	23-Jan-98	0.3
fair value of forward or futures	40.5895	



Stock Index Futures/Forwards

- Note that if the FV(Divs) that will be paid on the underlying stock portfolio is greater than the interest that can be earned on S , then the index futures price will be below the spot price.
- The forward rates, $h(t,T)$, can be locked in for borrowing and lending in perfect markets. (See Appendix C in Chapter 5.)



Implied Repo Rate (IRR)

- Given F and S , and the fact that $F=S(1+h)$, the periodic, or unannualized, implied repo rate is:

$$h(0,T) = \text{IRR} = \frac{F_0 - S_0}{S_0}$$

- With a carry return, $F = S + hS - \text{CR}$, and the periodic, or unannualized, implied repo rate is:

$$h(0, T) = \text{IRR} = \frac{F_0 + \text{CR} - S_0}{S_0}$$



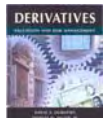
The Convenience Yield

- Non-carry commodities possess a **convenience yield**. That is, users will not sell their inventory (and buy forward), or sell short, because the good is needed in production.
- Thus, in the presence of a convenience yield, it is possible that $F \leq S + CC - CR$.
- That is, futures prices can be below their cost of carry theoretical price, and no reverse cash and carry arbitrage will occur.
- In the presence of a convenience yield, the spot-futures parity equation is written as: $F = S + CC - CR - \text{convenience return}$.



Backwardation = An Inverted Market

- When $F < S$, and futures prices decline, the longer the time until delivery, we say that the market is displaying **backwardation**, or that the market is **inverted**.
- Backwardation occurs when the convenience yield is high.



Do Forward Prices Equal Expected Future Spot Prices?

- Carry assets (gold, financials): NO! The forward price is **set** by the cost of carry model.
 - Stock index example. If there is a risk premium for owning stocks, how can $F = E(S_T)$?
- Physical commodities other than gold:
 - **unbiased expectations theory**: yes; $F_t = E_t(S_T)$.
 - **normal backwardation**: Hedgers are net short. Therefore, speculators are net long, and expect to earn a risk premium. As a result:
 $F_t < E_t(S_T)$.
 - **contango**: Hedgers are net long. Therefore, speculators are net short, and will not speculate unless they are rewarded. As a result:
 $F_t > E_t(S_T)$.



Forward Foreign Exchange Prices

- Define S as the spot price of a unit of FX (e.g., $S = \$0.00947/\text{JY}$).
- Define F as the forward price.
- Define h_{US} as the U.S. interest rate (e.g., $h_{\text{US}} = 5\%$)
- Define h_f as the interest rate in the foreign country (e.g., $h_f = 1.5\%$ in Japan).



Forward Foreign Exchange Prices

- Today:

borrow \$0.00947 to buy one unit of fx	+S	
buy one unit of fx	-S	+1FX
lend the one unit of fx		-1FX
sell $(1)(1+h_f)$ forward		
$CF_0 =$	0 0	

- At delivery (one year hence):

repay loan w/ interest (\$0.0099435)	-S(1+h _{US})
receive proceeds of FX loan (1.015JY)	
<u>sell FX proceeds (1.015JY) at forward price</u>	<u>F(1+h_f)</u>
$CF_T =$	$F(1+h_f) - S(1+h_{US})$



Interest Rate Parity Condition, I.

- Because $CF_T = F(1+h_f) - S(1+h_{US})$ must equal zero to preclude arbitrage, we conclude that the forward price must be:

$$F = S \frac{[1 + h_d(0, T)]}{[1 + h_f(0, T)]}$$

- In this example, $F = (0.00947)(1.05)/1.015 = \$0.00980/\text{JPY}$.
- For six months delivery, $F = (0.00947)(1.025)/1.0075 = \$0.00963449/\text{JPY}$.
- Note that F and S are both in terms of domestic/FX
- “The yen is at a forward premium to the dollar”.
- Of course, this holds for all exchange rates, not just \$/FX. That is, we can consider the forward price of euro in terms of yen (¥/€).



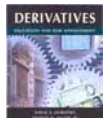
FinCad Result:

aaFXfwd		
FX spot - domestic / foreign	0.00947	
rate - domestic - annual	0.05	
rate - foreign - annual	0.015	
value (settlement) date	23-Sep-97	
forward delivery or repurchase date	23-Mar-98	
accrual method - domestic rate	4	30/ 360 ▼
accrual method - foreign rate	4	30/ 360 ▼
statistic	2	fair value of forward (domestic / foreign) ▼
fair value of forward (domestic / foreign)	0.009634491	



Interest Rate Parity Condition, II.

- This condition can be derived by thinking about the relationship as follows. U.S. investors must be indifferent between:
 - (a) lending \$ in the U.S. at the U.S. interest rate, and
 - (b) using the \$ to buy yen, lending yen in Japan at the Japanese interest rate, and selling the proceeds forward.



Forward Interest Rates

- Forward rates can be computed from the spot yield curve.
- Let $r(0,t_2)$ = the annual spot interest rate for the period ending at the end of the contract period (e.g., in a 1X7 FRA, this would be the 7-month rate).
- Let $r(0,t_1)$ = the annual spot interest rate for the period ending at the settlement date (e.g., the 1-month rate in a 1X7 FRA).
- The forward rate is $r(t_1,t_2)$. It is the six month rate, beginning one month hence. This is the contract rate of a FRA.



Forward Rates

- Let t_2 and t_1 be defined in fractions of a year.
- To compute the 6-month forward rate:

$$[1 + r(0, t_2)]^{t_2} = [1 + r(0, t_1)]^{t_1} [1 + r(t_1, t_2)]^{t_2 - t_1}$$

- So, if the 7-month rate is 6%, and the 1-month rate is 4.5%, then the six month forward rate, beginning one month hence, is computed as:

$$(1.06)^{0.5833} = (1.045)^{0.0833} (1+fr(1,7))^{0.5}.$$

- Therefore, $(1+fr(1,7))^{0.5} = 1.03078652$, and $fr(1,7) = 6.252\%$ (compounded), or 6.157% (simple).



FinCad Result:

aaFRAi		
value (settlement) date	23-Sep-97	
effective date	23-Oct-97	
terminating date	23-Apr-98	
FRA contract rate	0.035	
notional principal amount	1000000	
accrual method	4	30/ 360
discount factor curve	t_43_1	
interpolation method	1	linear
statistic	2	implied forward rate
t_43_1		
discount factor curve		
grid date	discount factor	
23-Sep-97	1	
23-Oct-97	0.996338645	
23-Apr-98	0.966580981	
implied forward rate	0.06157304	
note that $0.06157304/2 = 3.078652\%$		



Forward Rates

- Alternatively, in terms of an unannualized holding period return, $[h(t_1, t_2)]$:

$$[1 + h(0, t_2)] = [1 + h(0, t_1)][1 + fh(t_1, t_2)]$$

- In the previous example:

$$h(0, 7\text{-mo}) = (0.06)(7/12) = 0.035.$$

$$h(0, 1\text{-mo}) = (0.045)(1/12) = 0.00375.$$

- Thus, $1.035 = [1.00375][1 + fh(t_1, t_2)]$

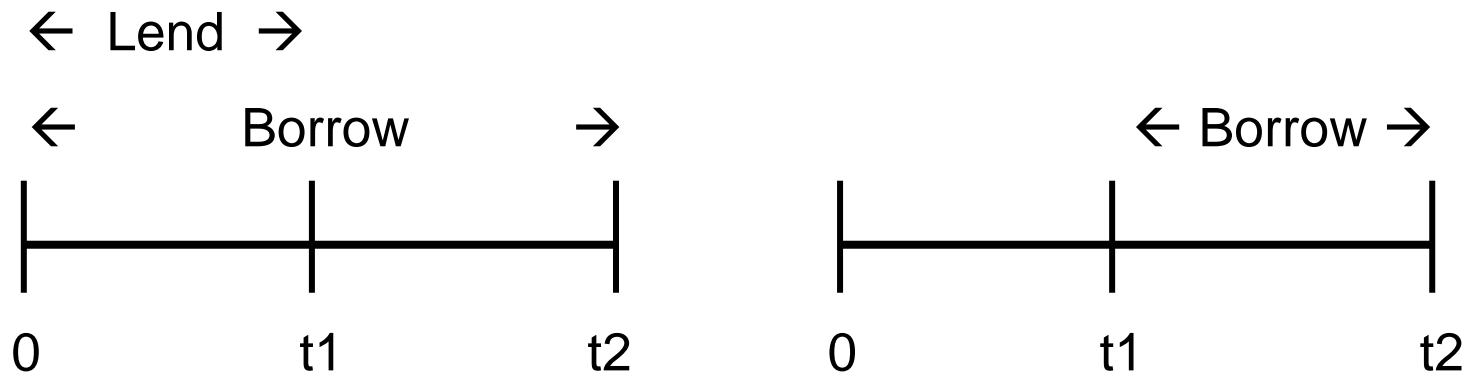
$$\text{and } fh(t_1, t_2) = fh(1, 7) = 0.0311325.$$

$$\text{annualize this to get } fh(1, 7) = 6.227\%$$



Forward Borrowing

- To lock in a borrowing rate from time t_1 to time t_2 , borrow $\$X$ from time 0 to time t_2 , and lend $\$X$ from time 0 to time t_1 :

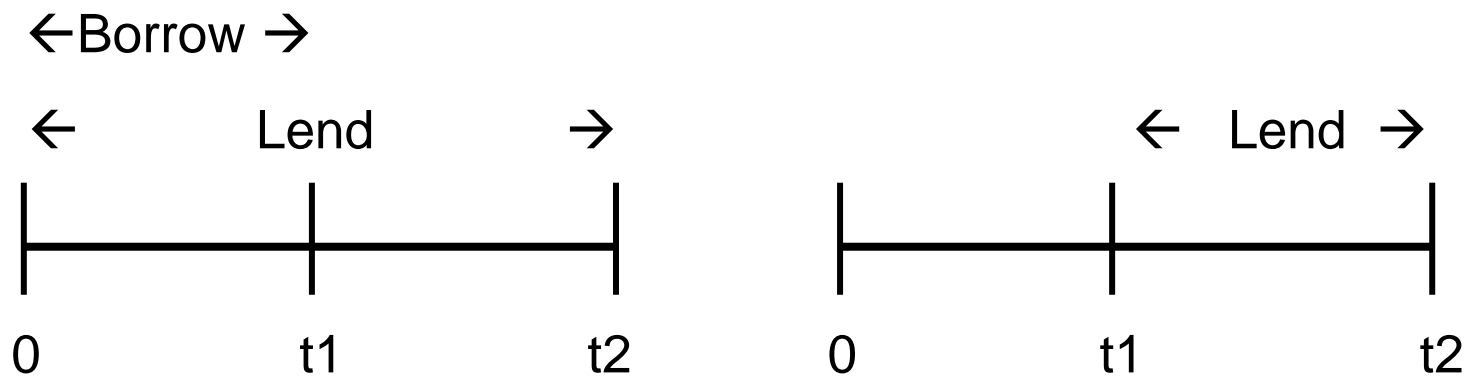


- These are reverse cash and carry trades: sell the deliverable asset short and lend the proceeds until the delivery date.



Forward Lending

- To lock in a lending rate from time t_1 to time t_2 , lend $\$X$ from time 0 to time t_2 , and borrow $\$X$ from time 0 to time t_1 :



- These are cash and carry trades: borrow to buy the deliverable asset.



Forward Lending, Example.

- Today:
 - borrow \$0.99626401 for one month at $r(0,1) = 4.5\%$
 - lend \$0.99626401 for seven months at $r(0,7) = 6\%$ (buy a seven month debt instrument)
- One month hence:
 - pay off your loan -\$1.00
- Seven months hence:
 - get repaid +\$1.03113325
- Result: today, you have locked in a forward holding period lending rate of 3.113325%, which is 6.227% annualized.



Outright versus Synthetic, Long Positions.

Outright Position: Equivalent Synthetic Position:

long forward/futures

borrow and buy spot

lend (i.e., buy Tbills)

buy spot and sell futures/
forwards

buy spot

buy futures/forwards and
lend



Outright versus Synthetic, Short Positions.

Outright Position: Equivalent Synthetic Position:

short forward/futures

sell spot and lend

Borrow (i.e.,sell Tbills)

sell spot and buy futures/
forwards

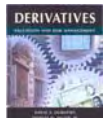
Sell spot

sell futures/forwards and
borrow



Some Extra Slides on this Material

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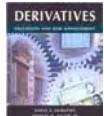
Arbitrage and the Cost of Carry Relationship

- Every commodity, financial asset, or service has a spot (or cash) price.
 - The prevailing market price for immediate delivery.
 - Example: Apples at Schnuck's Fine Foods.
- There may be more than one cash price for a commodity at a moment in time.
 - Heating Oil prices are quoted by region of the country.
- ***The Law of One Price.***



Arbitrage

- Definition: Arbitrage is the simultaneous purchase and sale of identical (or equivalent) assets in order to make a riskless profit in excess of the riskless rate of return.
- Exploits the fact that the same asset has a different price in different markets.
- Buy Low—Sell High.
- A **true** arbitrage:
 - Zero Investment
 - Zero Risk
 - Positive Return.
- Arbitrageur or Arbitrageuse?



Assertion: Investors always choose the least costly way of acquiring the spot asset.

- There are two ways to assure that you have an asset on the delivery day of a forward or futures contract:
 - 1. Go long a forward or futures contract today and hold it until delivery.
 - 2. Buy the asset in the cash market today and store it until the delivery date.
- These two methods of obtaining the spot asset results in the convergence of futures and spot prices on delivery day.



The Cost of Carry Relationship, AKA Spot Futures Parity

- This is an old time term.
- It refers to the costs associated with purchasing and ‘carrying’, i.e., “holding” an asset for a specified time.
- “Full-Carry” Futures (or Forward) Price Equals the Spot Price Plus Carrying Costs Minus Carry Returns.
- **IMPORTANT**: Why should this generally hold?



The Spot Futures Parity Equation

Standing at time 't':

$$FP^*(t,T) = S(t) + CC(t,T) - CR(t,T)$$

$FP^*(t,T)$ = THEORETICAL Futures Price at time t.

$S(t)$ = Spot Price at time t.

$CC(t,T)$ = Carrying Costs from time t to time T.

$CR(t,T)$ = Carry Returns from time t to time T.

NB: $FP(t,T)$ is the OBSERVED Futures Price at time t.



Using the Cost or Carry Relationship, I.

- Suppose you observe the following:
 - The spot price of gold is \$280/oz.
 - The annual risk-less borrowing and lending rate is 10%.
 - The observed gold futures price is \$300/oz.
- The futures contract expires in 6 months.
- There are no other carrying costs or carry returns.



Using the Cost or Carry Relationship, II.

- How do you use this information?
- First, calculate the “Full-Carry” Futures Price.

$$\text{FCFP} = 280 + [(180/360) \times 0.10 \times 280] = 294$$

- Then, compare FCFP to the observed, futures price [294 versus 300].
- Hmm... the observed futures price is **higher** than the full carry futures price.

Quickly!! What to do, what to do?



The 'Cash and Carry' Arbitrage, I.

- Today:
 - Borrow \$280 at 10% for six months.
 - Buy gold in the spot market for \$280, and store it (i.e., “carry” it.)
 - Sell a futures contract, with a futures price of \$300.



The 'Cash and Carry' Arbitrage, II.

- In Six Months, there is a “risk-less” profit of $\$300 - 294 = \6 , *with zero initial investment*.
 - Deliver the gold against the short futures position, as agreed.
 - “Receive” a net \$300 (through the marking to market mechanism).
 - Repay loan amount of \$294.

$$280 + [(180/360) \times 0.10 \times 280] = 294$$



The 'Cash and Carry' Arbitrage, III.

- Note Bene:
 - The key to this risk-less strategy is that the cash flows are known today.
 - Arbitrage involving futures contracts is *not completely risk-less* (because of the marking to market interim cash flows).
 - However, academic studies have found this risk to be very small.



Using the Cost or Carry Relationship, III.

- Suppose you observe the following:
 - The spot price of gold is \$280/oz.
 - The annual risk-less borrowing and lending rate is 10%.
 - The observed gold futures price is \$290/oz.
- The futures contract expires in 6 months.
- There are no other carrying costs or carry returns.
- ***Gold can be sold short.***
- ***The full amount of the short-sale proceeds is available to invest at the risk-less interest rate.***



Using the Cost or Carry Relationship, IV.

- How do you use this information?
- First, calculate the “Full-Carry” Futures Price.

$$\text{FCFP} = 280 + [(180/360) \times 0.10 \times 280] = 294$$

- Then, compare FCFP to the observed, futures price [294 versus 300].
- Hmm... the observed futures price is **lower** than the full carry futures price.

Quickly!! What to do, what to do?



The 'Reverse Cash and Carry' Arbitrage, I.

- Today:
 - Sell gold SHORT in the spot market for \$280.
 - Invest the \$280 at 10% for six months.
 - Buy a futures contract, with a futures price of \$290.



The 'Reverse Cash and Carry' Arbitrage, II.

- In Six Months, there is a “risk-less” profit of \$294 – 290 = \$4, *with zero initial investment*.
 - Deliver the gold against the short spot position, as agreed.
 - “Pay” a net \$290 (through the marking to market mechanism) and take delivery of the gold (which you will use to repay the short-sale in the spot).
 - Receive \$294 from the investment:

$$280 + [(180/360) \times 0.10 \times 280] = 294$$



Chapter 14

Introduction to Options

- Make sure that you review the ‘options’ section from Chapter 1. We will not spend too much time on the slides whose titles begin with “Recall:”



Recall: Options

- Option Contracts Separate **Obligations** from **Rights**.
- Two basic option types:
 - Call options
 - Put options
- Two basic option positions:
 - Long
 - Short (write)



Recall: Call Option Contracts

- A call option is a contract that gives the **owner** of the call option the **right, but not the obligation, to buy** an underlying asset, at a fixed price ($\$K$), on (or sometimes before) a pre-specified day, which is known as the expiration day.
- The **seller** of a call option, the call **writer**, is **obligated** to deliver, or **sell**, the underlying asset at a fixed price, on (or sometimes before) expiration day (T).
- The fixed price, K , is called the *strike price*, or the *exercise price*.
- ***Because they separate rights from obligations, call options have value.***
- We denote the call premium as “C”.



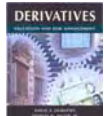
“Moneyness”: In-the-money, out-of-the-money, and at-the-money

- Define S as the price of the underlying asset, and K as the strike price. Then, for a call:
 - In-the-money, if $S > K$
 - Out-of-the-money, if $S < K$
 - At-the-money, if $S \sim K$
 - Deep-in-the-money, if $S \gg K$
 - Deep-out-of-the-money, if $S \ll K$



Intrinsic Value and Time Value

- Intrinsic value of a call = $\max(0, S-K)$
 - (You read this as: “The maximum of: zero OR the stock price minus the strike price.”)
- Time value = $C - \text{intrinsic value}$
- Time value declines as the expiration date approaches. At expiration, time value = 0.

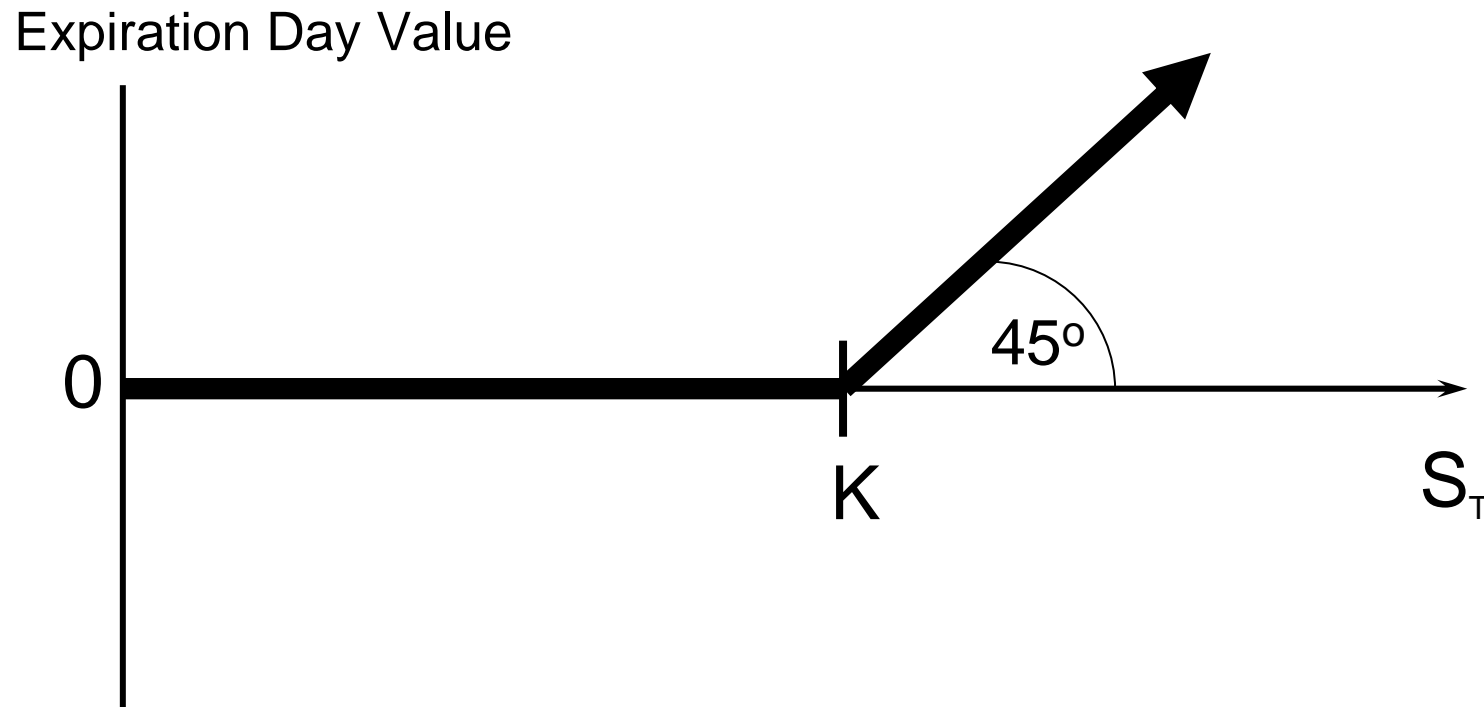


Example: Intrinsic Value for a Call

- **Suppose a call option is selling for \$1.70. The underlying asset price is \$41.12.**
 - Consider a call with a strike price of 40. Is this call in the money or out of the money? Calculate the intrinsic value of this call. What is the time value?
 - Consider a call with a strike price of 45. Is this call in the money or out of the money? Calculate the intrinsic value of this call. What is the time value?

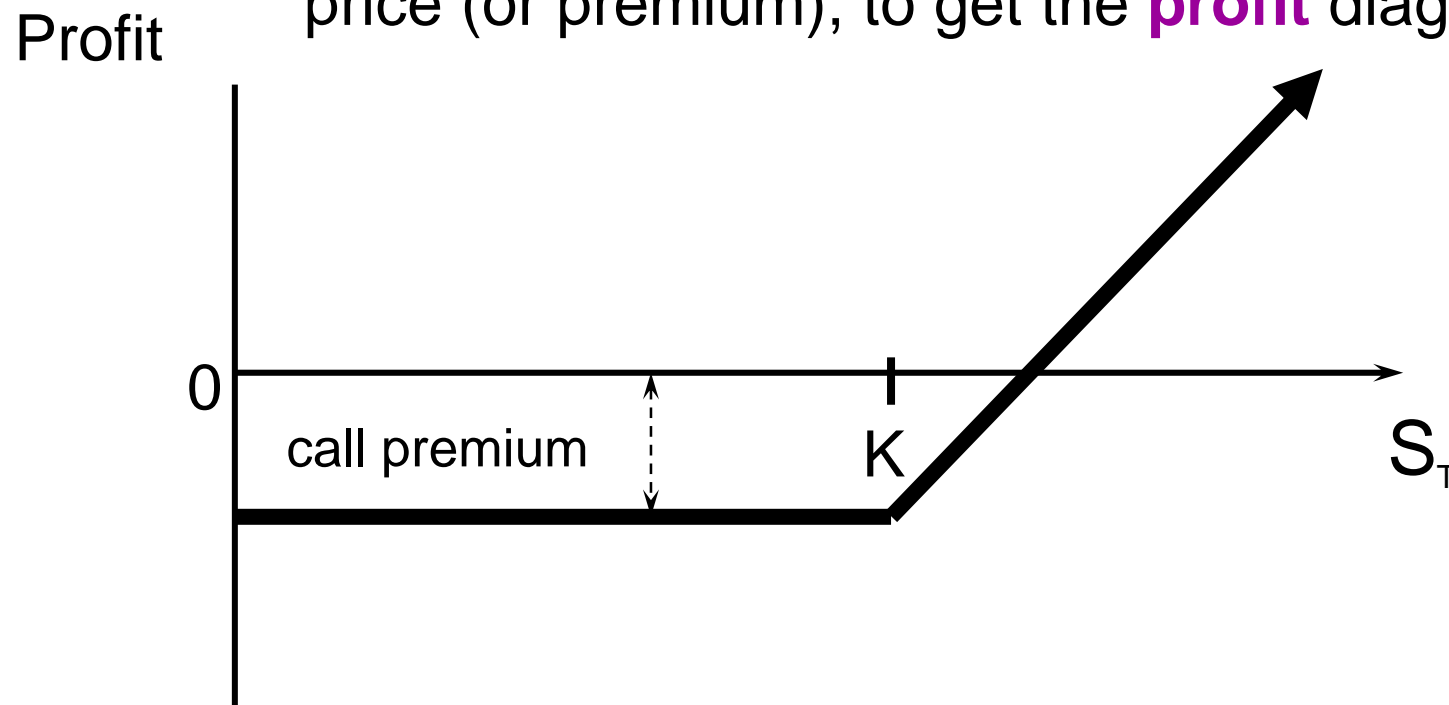


Recall: **Payoff** Diagram for a Long Call Position, at Expiration

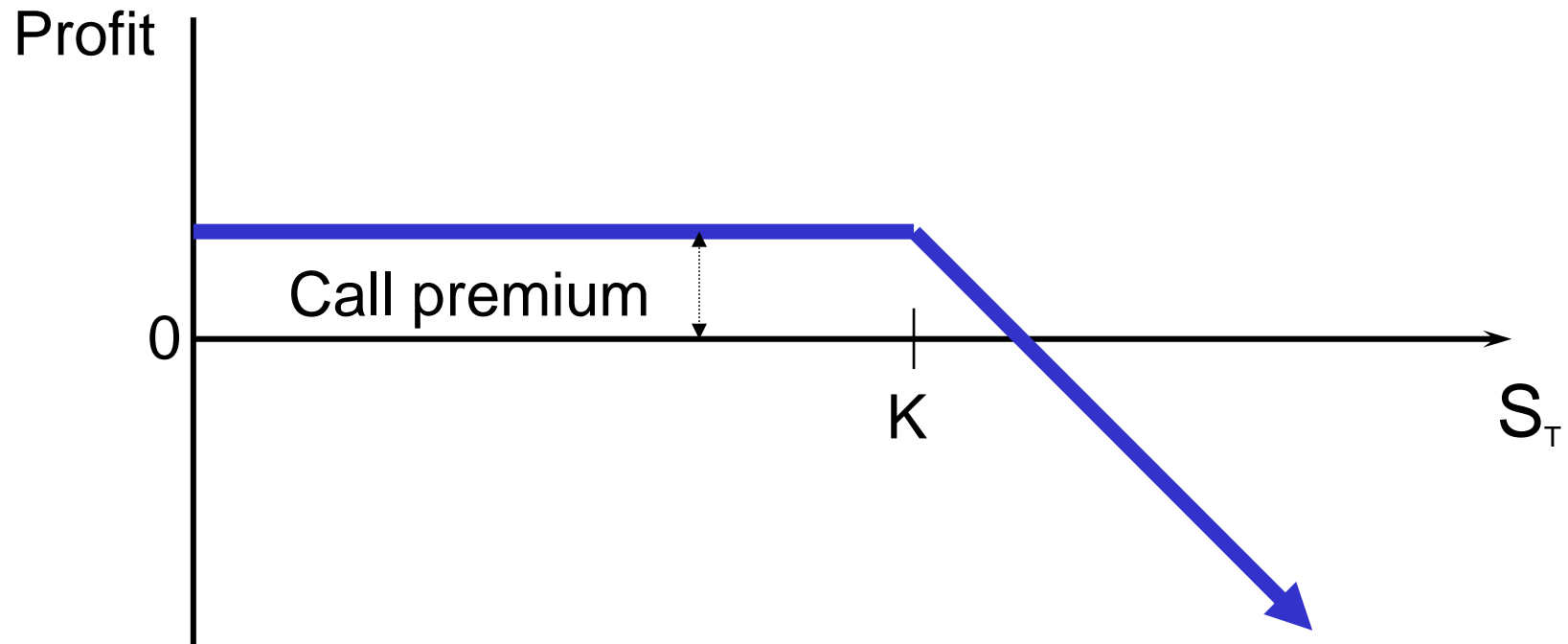


Recall: Profit Diagram for a Long Call Position, at Expiration

We lower the **payoff** diagram by the call price (or premium), to get the **profit** diagram



Recall: Profit Diagram for a Short Call Position, at Expiration



Recall: Put Option Contracts

- A put option is a contract that gives the **owner** of the put option the **right, but not the obligation, to sell** an underlying asset, at a fixed price, on (or sometimes before) a pre-specified day, which is known as the expiration day (T).
- The **seller** of a put option, the put **writer**, is **obligated** to take delivery, or **buy**, the underlying asset at a fixed price ($\$K$), on (or sometimes before) expiration day.
- The fixed price, K , is called the *strike price*, or the *exercise price*.
- ***Because they separate rights from obligations, put options have value.***
- The put premium is denoted “P”.



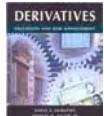
Put Option “**Money**ness”

- Define S as the price of the underlying asset, and K as the strike price.
- Then, for a put option:
 - In-the-money, if $K > S$
 - Out-of-the-money, if $K < S$
 - At-the-money, if $K \sim S$
 - Deep-in-the-money, if $K \gg S$
 - Deep-out-of-the-money, if $K \ll S$
- Intrinsic value of a put = $\max(0, K-S)$
- Time value = P - intrinsic value

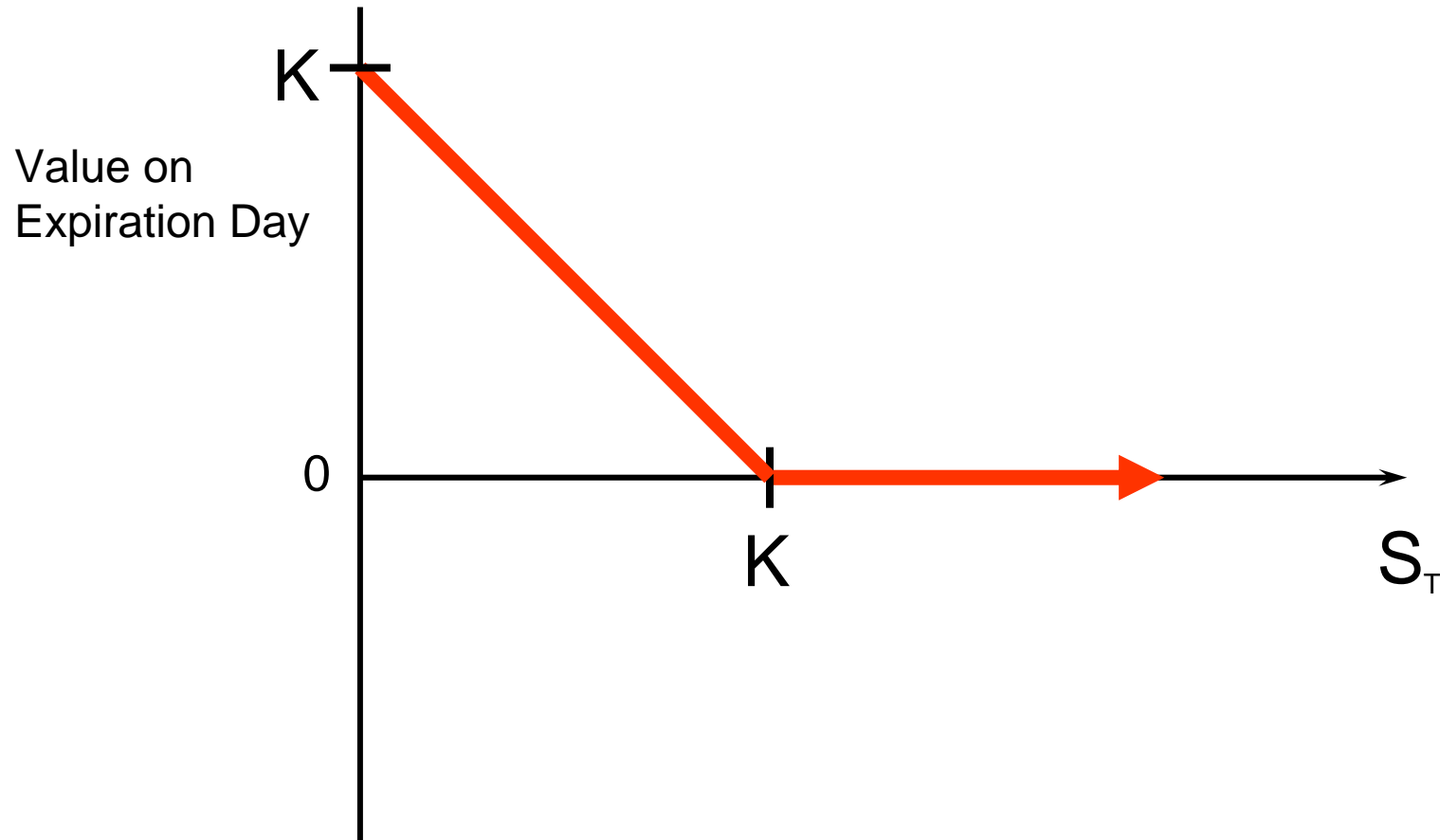


Example: Intrinsic Value for a Put

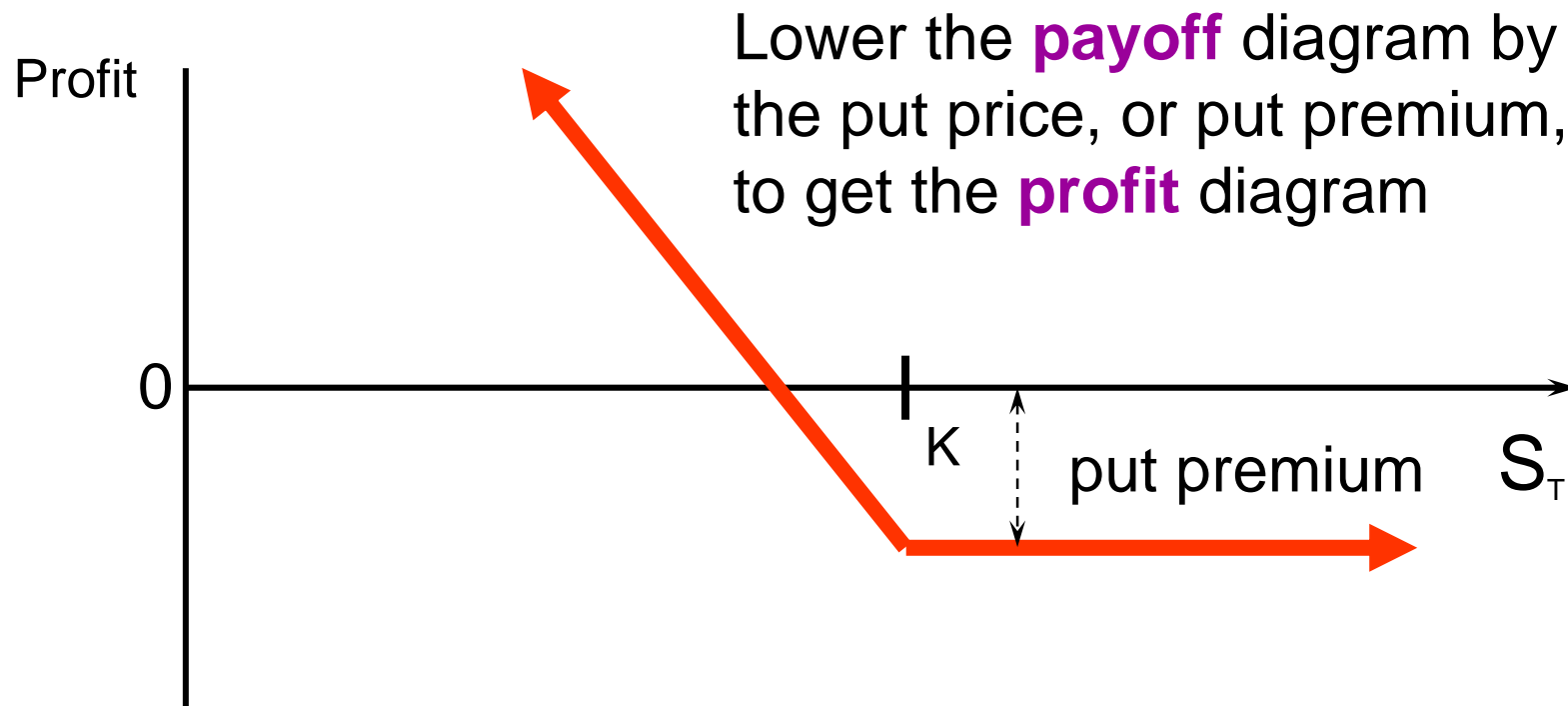
- **Suppose a put option is selling for \$5.70. The underlying asset price is \$41.12.**
 - Consider a put with a strike price of 40. Is this put in the money or out of the money? Calculate the intrinsic value of this put. What is its time value?
 - If the put has a strike price of 45, then is it in the money or out of the money? Calculate the intrinsic value of a put with a strike price of 45. What is its time value?



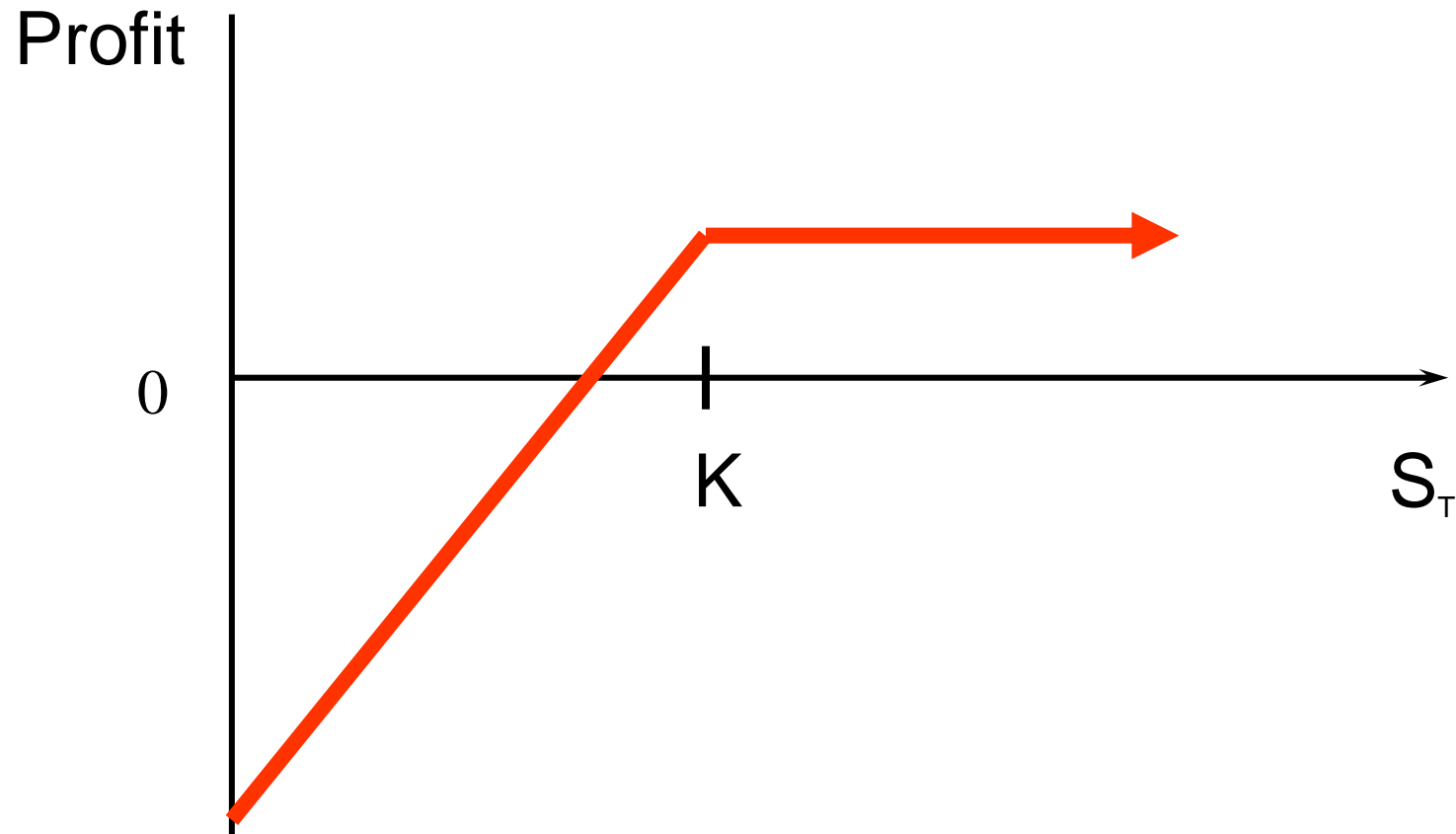
Recall: **Payoff** diagram for a long put position, at expiration



Recall: **Profit** Diagram for a Long Put Position, at Expiration

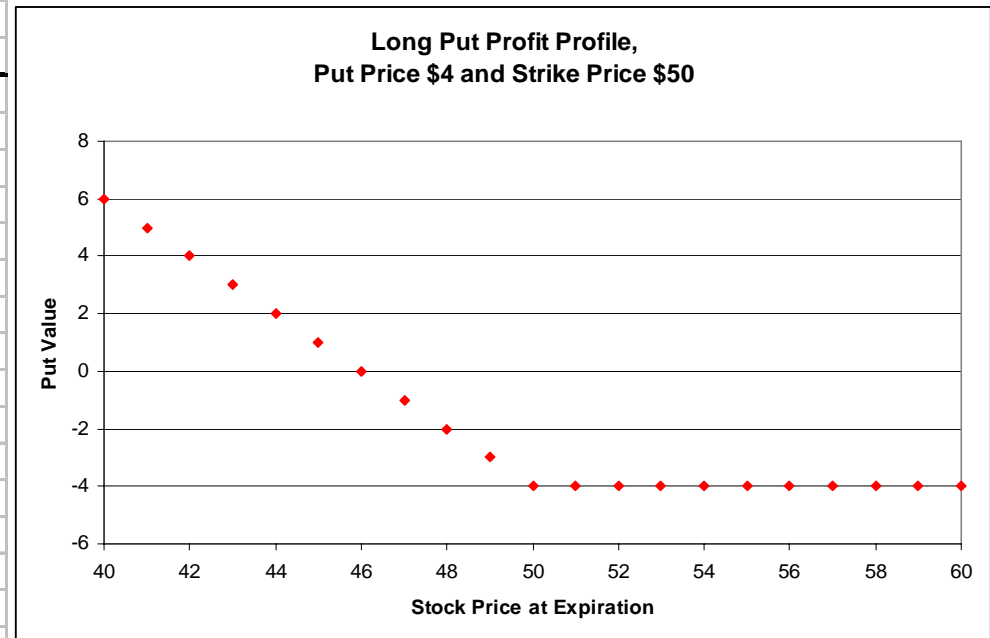


Recall: Profit Diagram for a Short Put Position, at Expiration

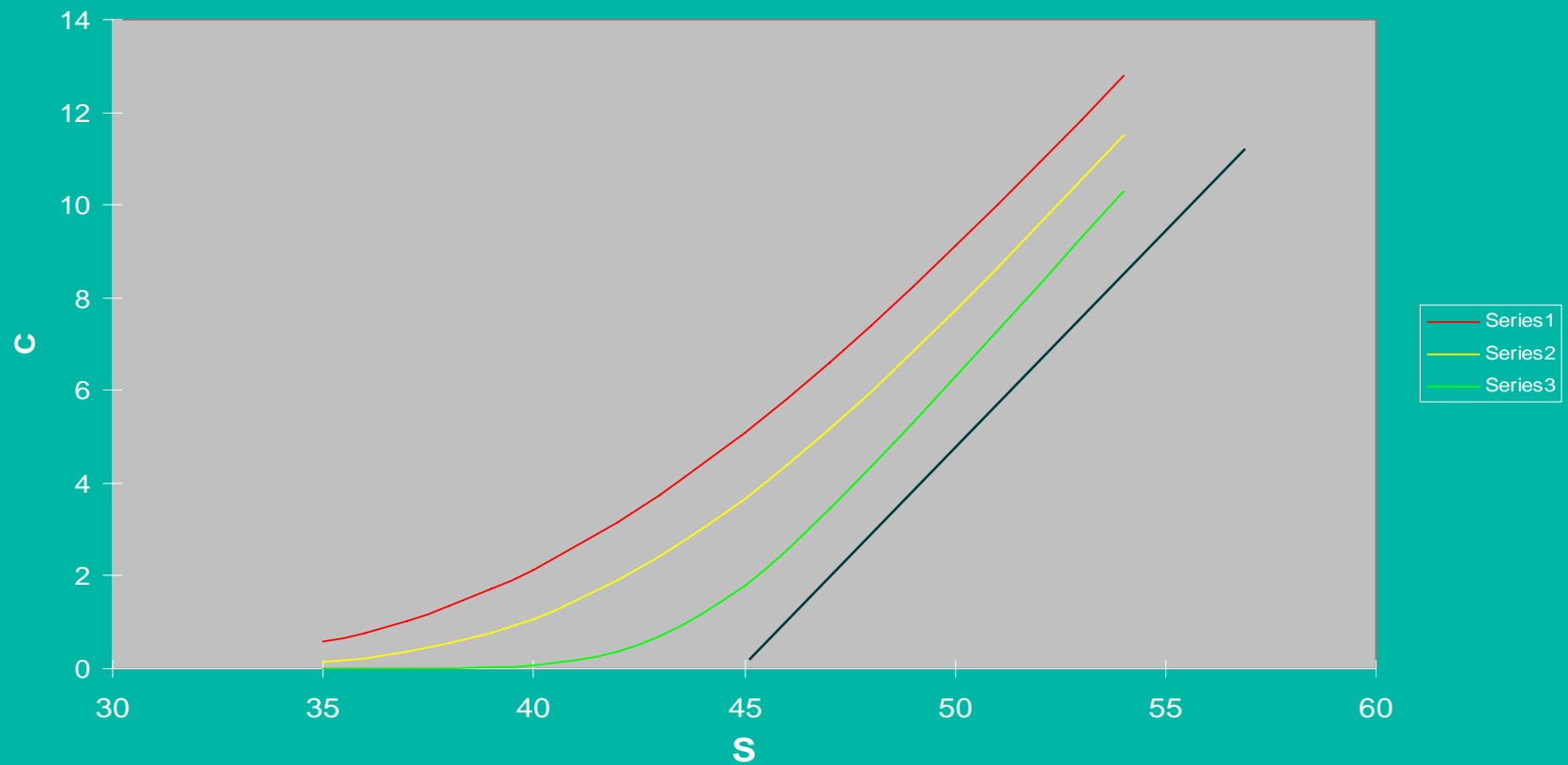


Let $K=50$; $P=4$

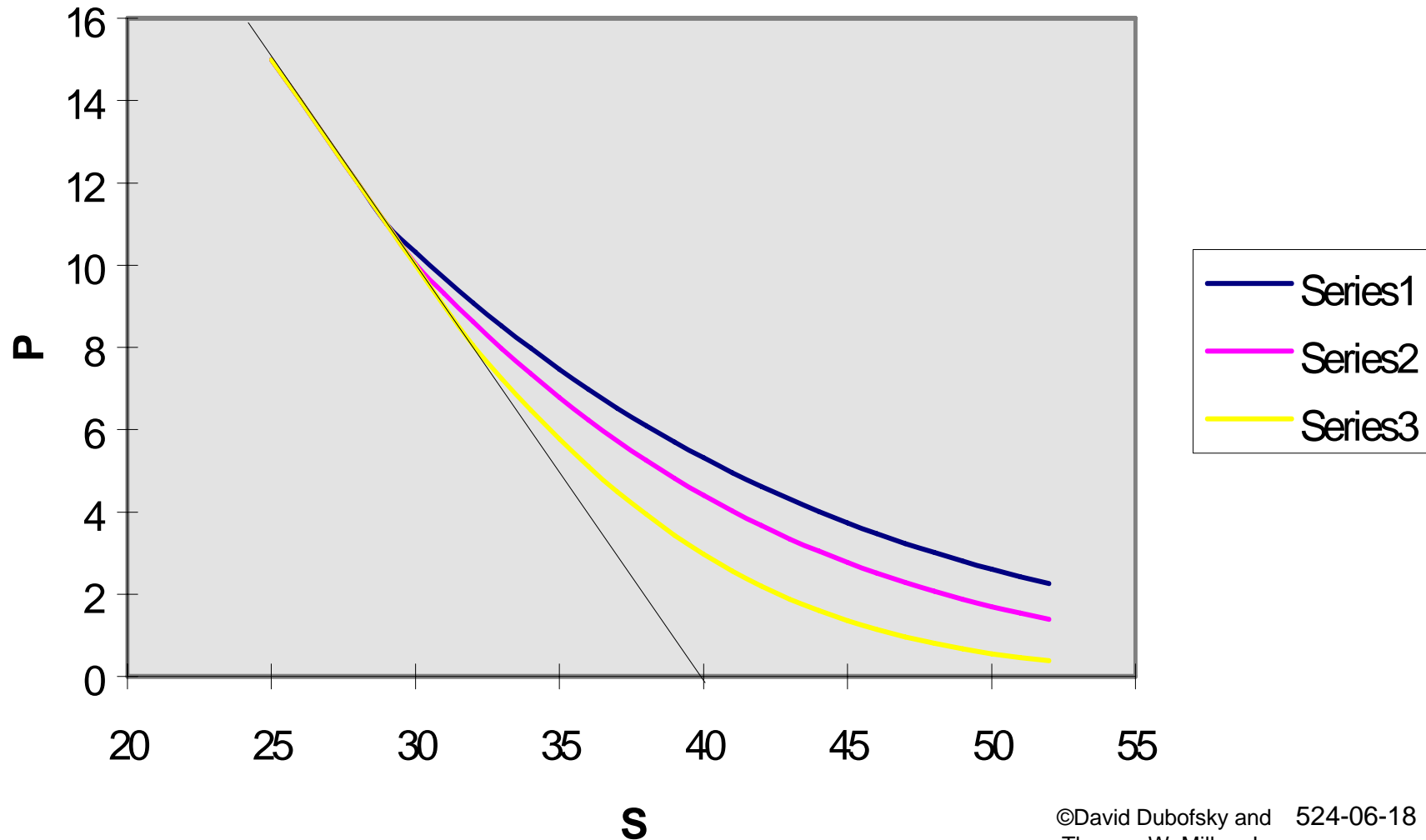
Stock Price at Expiration	Intrinsic Value of Put	Cost of Put Option	Position Profit
40	10	4	6
41	9	4	5
42	8	4	4
43	7	4	3
44	6	4	2
45	5	4	1
46	4	4	0
47	3	4	-1
48	2	4	-2
49	1	4	-3
50	0	4	-4
51	0	4	-4
52	0	4	-4
53	0	4	-4
54	0	4	-4
55	0	4	-4
56	0	4	-4
57	0	4	-4
58	0	4	-4
59	0	4	-4
60	0	4	-4



Call Pricing Prior to Expiration



Put Pricing Prior to Expiration



Comparative Statics

All else equal:

Call values rise as

- S rises
- lower K
- longer T
- higher volatility
- higher r

Puts rise as

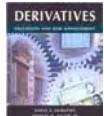
- S falls
- higher K
- ??????
- higher volatility
- lower r

- American put values rise with a longer T
- European put values are indeterminate with respect to T



Reading Option Price Data

- See WSJ, and <http://quote.cboe.com/QuoteTable.asp>
- Options on individual stocks
 - Leaps
- Index options (& leaps)
- Futures Options
- FX Options (see <http://www.phlx.com/products/currency.html>)



Index Options

- Most index options are European.
- Index options are cash settled.
 - At expiration, the owner of an in the money call receives $100 \times (S_T - K)$ from the option writer.
 - At expiration, the owner of an in the money put receives $100 \times (K - S_T)$ from the option writer.
 - Equivalently, the option owner receives its intrinsic value on the expiration day.



Futures Options

- The owner of a call on a futures contract has the right to go long a futures contract at the strike price.
- The exerciser of a call on a futures contract goes long the futures contract, which is immediately marked to market (he receives $F - K$). The writer of that call must pay the intrinsic value and either a) deliver the futures contract he owns, or b) go short the futures contract.
- The exerciser of a put on a futures contract goes short the futures contract, which is immediately marked to market (she receives $K - F$). The writer of that put must pay the put's intrinsic value and either a) has the obligation to assume a long position in the futures contract, or b) if she was short the futures to begin with, she will see her futures position offset.



Other Interesting Options

- Flex Options (<http://www.cboe.com/Institutional/Flex.asp>)
- Interest Rate Options (mostly OTC, but see Barrons, and http://www.cboe.com/OptProd/understanding_products.asp#irate and <http://www.cboe.com/common/pageviewer.asp?sec=4&dir=opprodspec&file=i-rateop.doc> Ticker symbols are IRX, FVX, TNX, and TYX)
- Exotic Options; see chapter 20
 - Asian Options ($C(T) = S(\text{AVG}) - K$)
 - Lookback Options ($C(T) = S(T) - \text{MIN}(S)$)
 - Chooser options ($\text{ChO}(T) = \max(c, p)$)
 - Etc.
- Swaptions (section 20.2.5)



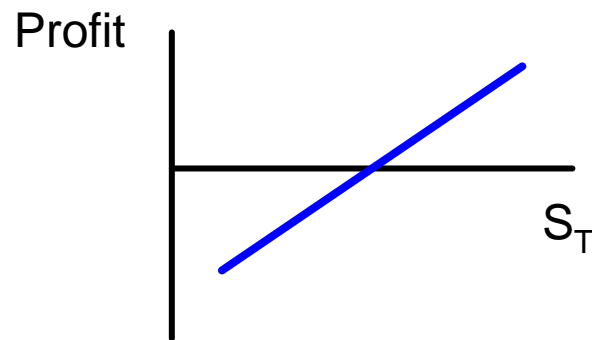
Chapter 15

Option Strategies and Profit Diagrams

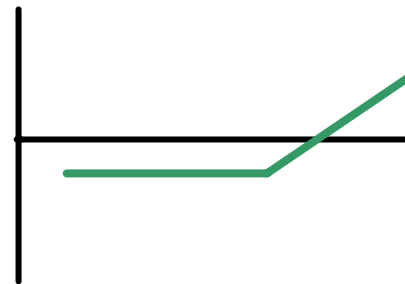
- In the diagrams that follow, it is important to remember that the diagrams that follow are based on option intrinsic value, at expiration.
- **Helpful Hint: In the diagrams that follow, the 'KINKS' are at strike prices.**
- **Throughout this chapter, bid-ask spreads and brokerage fees are assumed to be zero.**
- **$C_T = \max(0, S_T - K)$ and $P_T = \max(0, K - S_T)$**



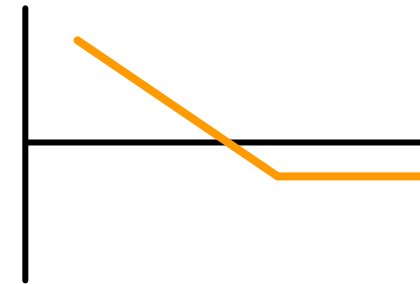
Quick Quiz: Identify These Six Basic Derivative Positions:



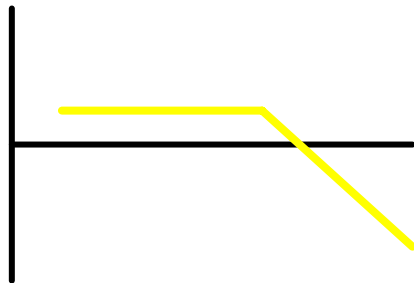
[A]



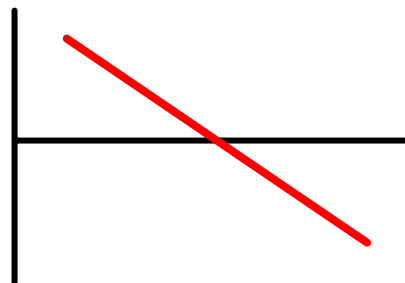
[B]



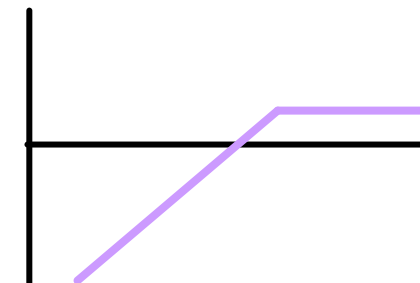
[C]



[D]



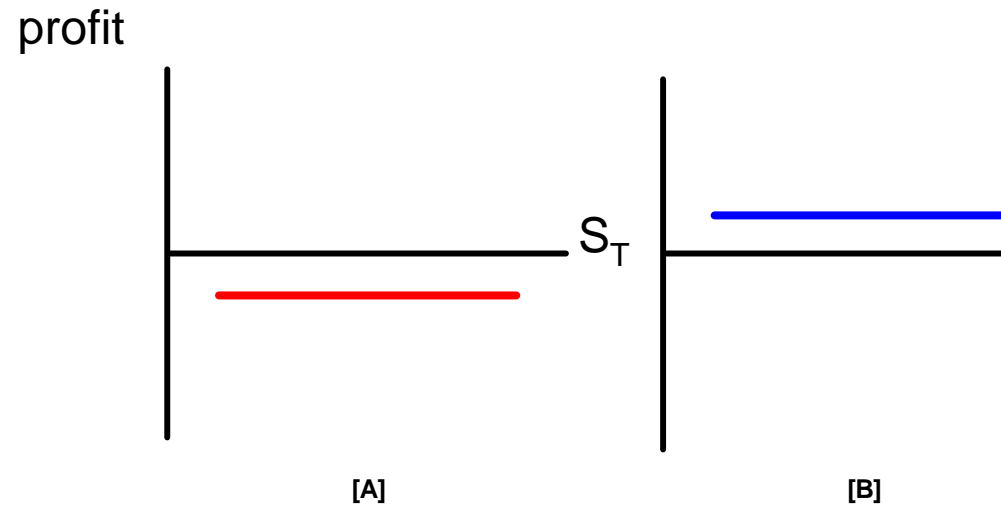
[E]



[F]



To These Six Basic Positions, add These Two Riskless Positions



- Why are these positions riskless?
- What do they represent?
 - Riskless Borrowing, [A], Receive money today, always pay money at ‘expiration’ of the loan. **(AKA: Short (sell) T-bills)**
 - Lending, [B], Pay money today, always receive money at ‘expiration’ of the loan. **(AKA: Long (buy) T-bills)**



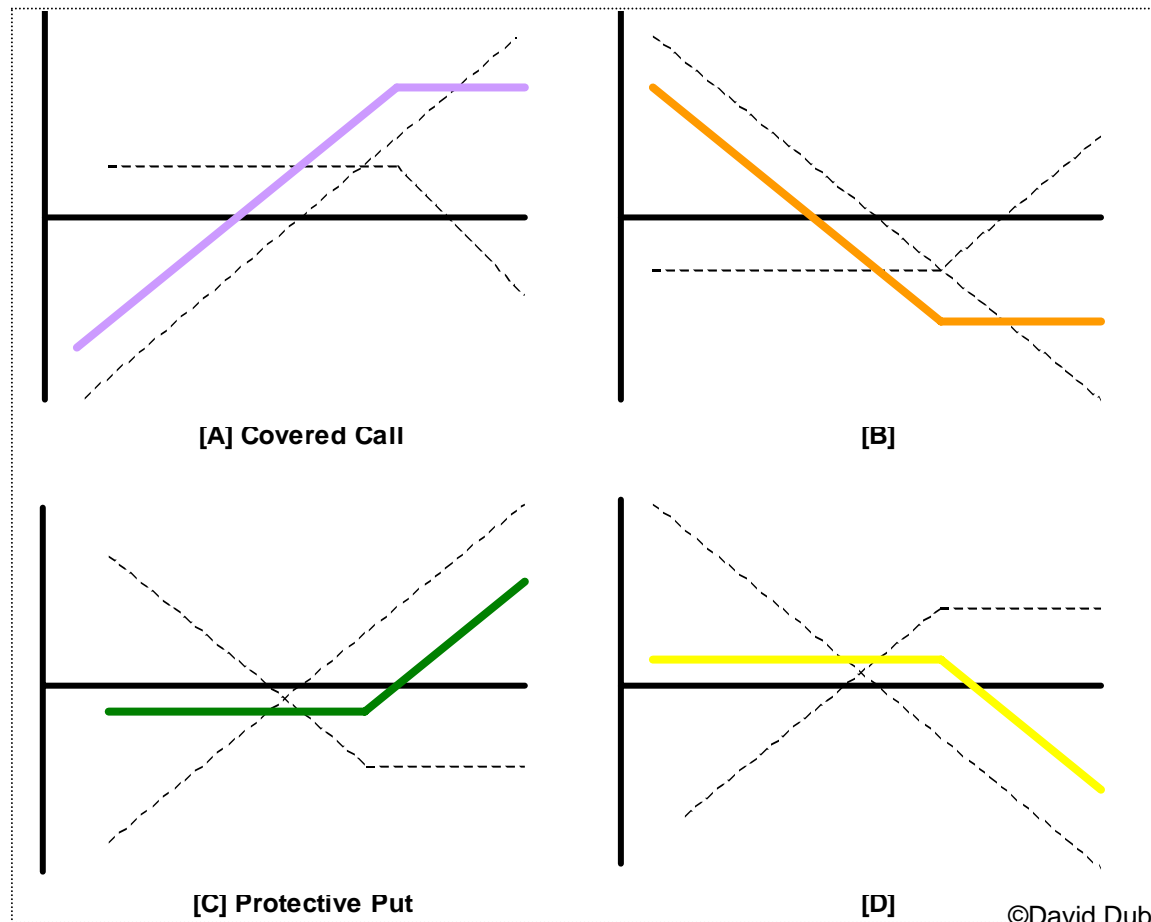
There are Three Basic Option Trading Strategies

- Take a position in an option and the underlying.
- Take a position in 2 or more options of the same **type (This is called a spread)**
 - Same **type** means:
 - Use only calls –or–
 - Use only puts
- Take a position in a **mixture** of calls and puts **(This is called a combination.)**



Positions in an Option and the Underlying

- Try to identify the positions in the option, the underlying, and the net position.
- NB: The 'KINKS' occur at strike prices.

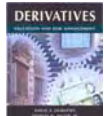


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How to construct a **profit table**

- Begin by computing the initial outlay, CF_0 .
- Choose a range of day T prices for the underlying asset (begin at ~\$2 below the lowest strike price and end at ~\$2 above the highest strike price)
- Compute the expiration day payoffs for each position if they are offset on day T
- Add up the expiration day payoffs from offsetting each position; this gives you CF_T .
- Add CF_T to the initial outlay, CF_0 , and you will have computed the strategy profit for each relevant value of S_T .



Example: Protective Put, I.

- Suppose you currently own 100 shares of a stock, with a value of \$86.38/share.
- You fear it may fall in value in the short run, but do not want to sell now.
- You see the following option data:

Strike	Call	Put
75	11.50	0.75
80	7.00	1.38
85	4.25	3.25
90	2.25	6.13
95	0.81	8.88

- You decide to purchase an 85 put.
- The protective put strategy is long stock + long put.



Example: Protective Put, II

- That is:

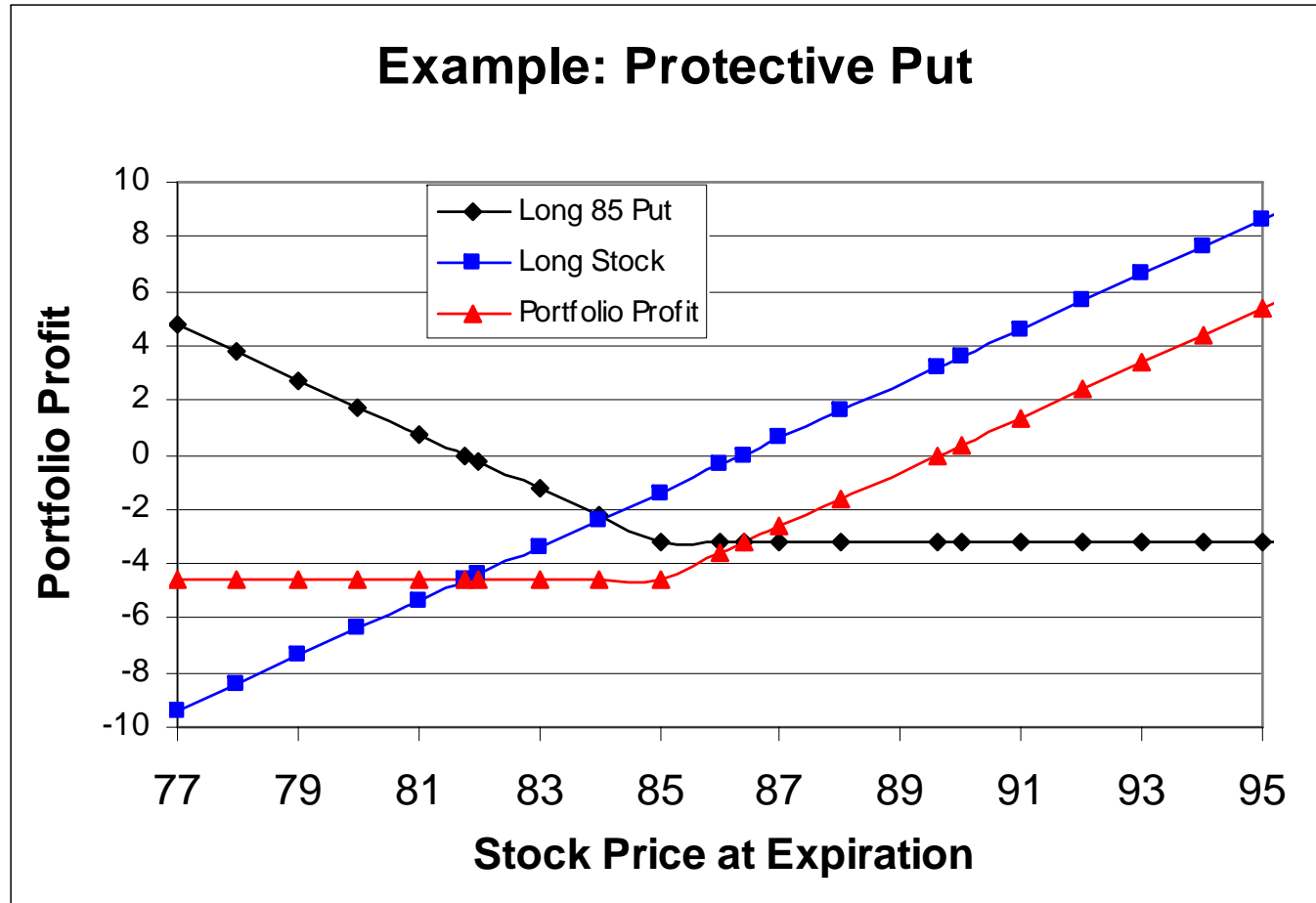
<u>At time 0</u>	
Buy stock	-86.38
<u>Buy put</u>	<u>- 3.25</u>
CF(0)	-89.63

Stock Price at Expiration	P(T) 85 Put	Sell stock	CF(T)	CF(0)	CF(0)+CF(T) Portfolio Profit
78.00	7.00	78.00	85.00	-89.63	(4.63)
79.00	6.00	79.00	85.00	-89.63	(4.63)
80.00	5.00	80.00	85.00	-89.63	(4.63)
81.00	4.00	81.00	85.00	-89.63	(4.63)
81.75	3.25	81.75	85.00	-89.63	(4.63)
82.00	3.00	82.00	85.00	-89.63	(4.63)
83.00	2.00	83.00	85.00	-89.63	(4.63)
84.00	1.00	84.00	85.00	-89.63	(4.63)
85.00	0.00	85.00	85.00	-89.63	(4.63)
86.00	0.00	86.00	86.00	-89.63	(3.63)
86.38	0.00	86.38	86.38	-89.63	(3.25)
87.00	0.00	87.00	87.00	-89.63	(2.63)
88.00	0.00	88.00	88.00	-89.63	(1.63)
89.25	0.00	89.25	89.25	-89.63	(0.38)
89.63	0.00	89.63	89.63	-89.63	0.00
90.00	0.00	90.00	90.00	-89.63	0.37
91.00	0.00	91.00	91.00	-89.63	1.37
92.00	0.00	92.00	92.00	-89.63	2.37

This is the range of S(T) that you really need



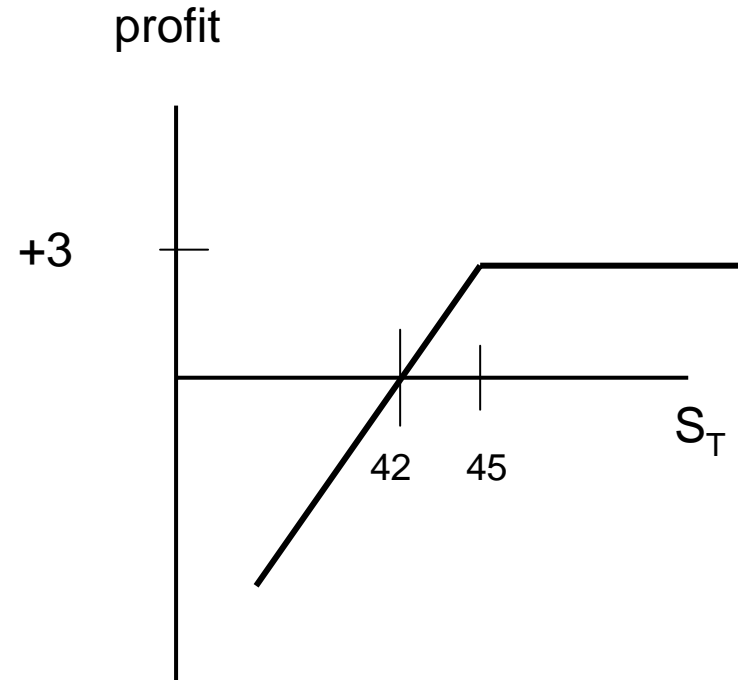
Then, One Can Plot the Constituent Profits and the Portfolio Profits



Writing a covered call

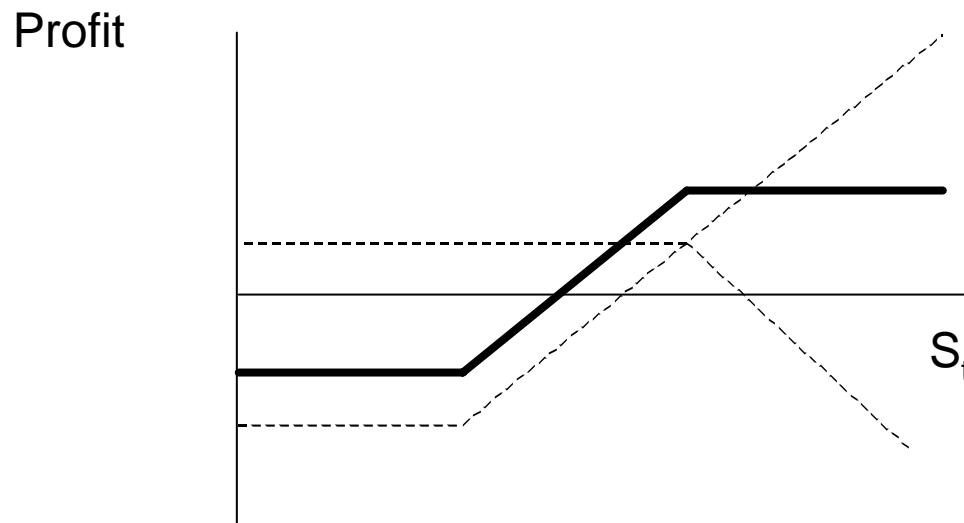
- Buy a stock for $S(0) = 43$
- Sell a call with $K = 45$ for $C(0) = 1$
- Initial outlay is -42

Stock Price at Expiration	offset C(T) 45 call	Sell stock	CF(T)	CF(0)	CF(0)+CF(T) Portfolio Profit
40.00	0.00	40.00	40.00	-42.00	(2.00)
41.00	0.00	41.00	41.00	-42.00	(1.00)
42.00	0.00	42.00	42.00	-42.00	0.00
43.00	0.00	43.00	43.00	-42.00	1.00
44.00	0.00	44.00	44.00	-42.00	2.00
45.00	0.00	45.00	45.00	-42.00	3.00
46.00	-1.00	46.00	45.00	-42.00	3.00
47.00	-2.00	47.00	45.00	-42.00	3.00
48.00	-3.00	48.00	45.00	-42.00	3.00



Vertical Spreads, I.

- [A] Bullish Vertical Spread with Calls (AKA: A Bull Call Spread, or a bullish call money spread).
 - Buy Call with lower strike.
 - Sell Call with higher strike.



[A] Bull Call Spread

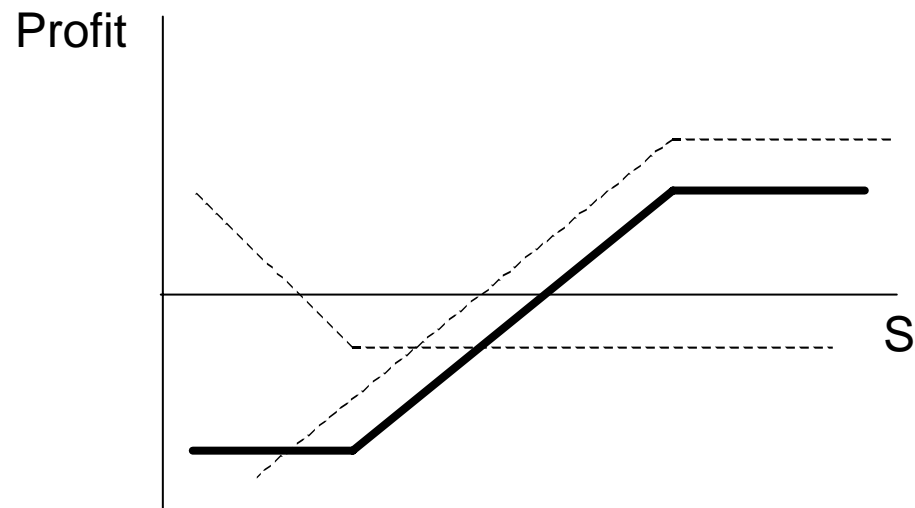
Identify the Strike Prices
Using the 'kinks'

Note that there is an initial **outlay** with this strategy; the purchased call has a higher price than the written call



Vertical Spreads, II.

- [B] Bullish Vertical Spread with Puts (AKA: A Bull Put Spread.)
 - Buy Put with lower strike.
 - Sell Put with higher strike.



[B] Bull Put Spread

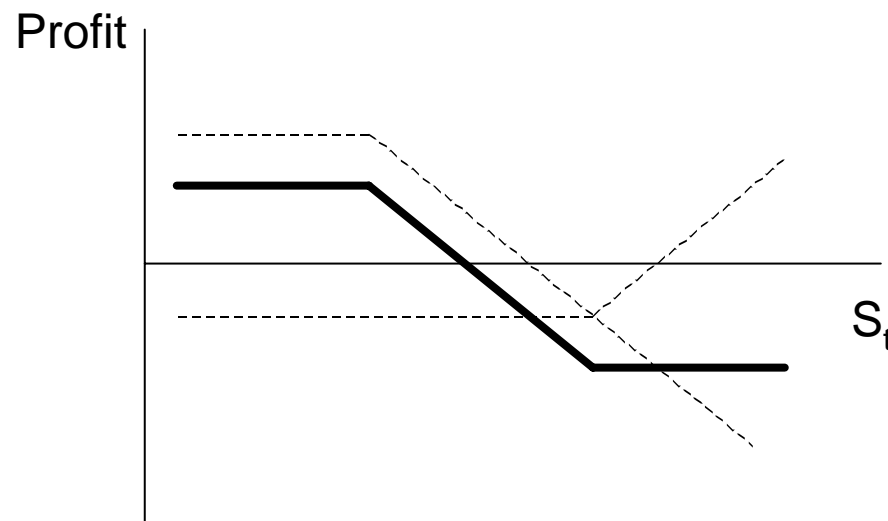
There is an initial cash **inflow** with this strategy.

Again: Identify the Strikes by the 'Kinks'. Do they make sense?



Vertical Spreads, III.

- [C] Bearish Vertical Spread with Calls (AKA: A Bear Call Spread.)
 - Buy call with higher strike.
 - Sell call with lower strike.



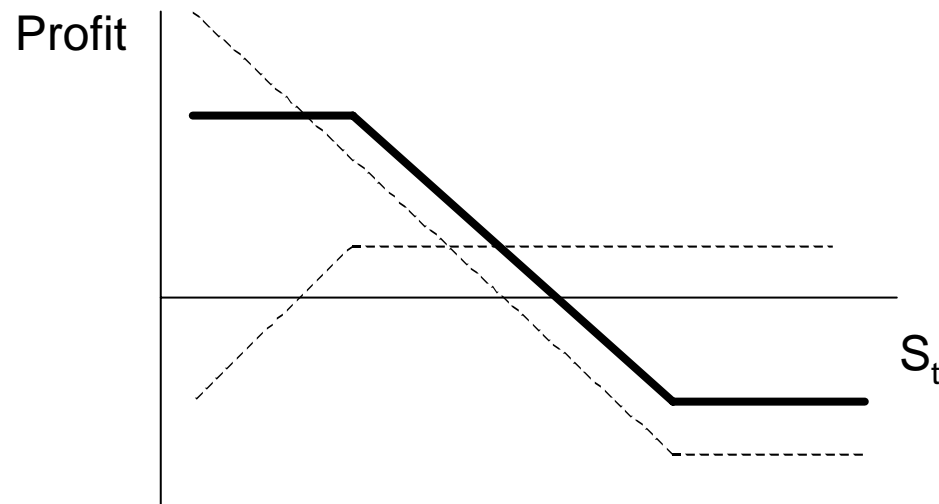
[C] Bear Call Spread

Is there an initial cash inflow or outflow?



Vertical Spreads, IV.

- [D] Bearish Vertical Spread with Puts (AKA: A Bear Put Spread.)
 - Buy put with higher strike.
 - Sell put with lower strike.



Is there an initial cash inflow or cash outflow?

[D] Bear Put Spread



Example: Bullish Vertical Spread with Calls, I.

- Suppose you observe the following data from the CBOE:
 - Price of Jan 80 Call: \$3.75 (\$375 per contract)
 - Price of Jan 75 Call: \$5.00 (\$500 per contract)
- You decide to buy the Jan 75 call and sell the Jan 80 Call.
- Today, your outlay is \$1.25, or \$125 per contract.
- At expiration:
 - At any price lower than \$75, your payoff is \$0 and your loss is \$1.25 (your initial outlay).
 - If the underlying price is \$76 at expiration, your payoff is \$1.00, and your loss ($CF_0 + CF_T$) is \$0.25.
 - If the underlying price is \$77 at expiration, your payoff is \$2.00, and your profit is \$0.75.
 - If the underlying price is \$79 at expiration, your payoff is \$4.00, and your profit is \$2.75.
 - At any price equal to or above \$80, your payoff is \$5.00, or \$500, and your profit is 3.75.



Example: Bullish Vertical Spread with Calls, II.

- Today: Buy Jan 75 call -5
Sell Jan 80 call +3.75
 CF(0) -1.25

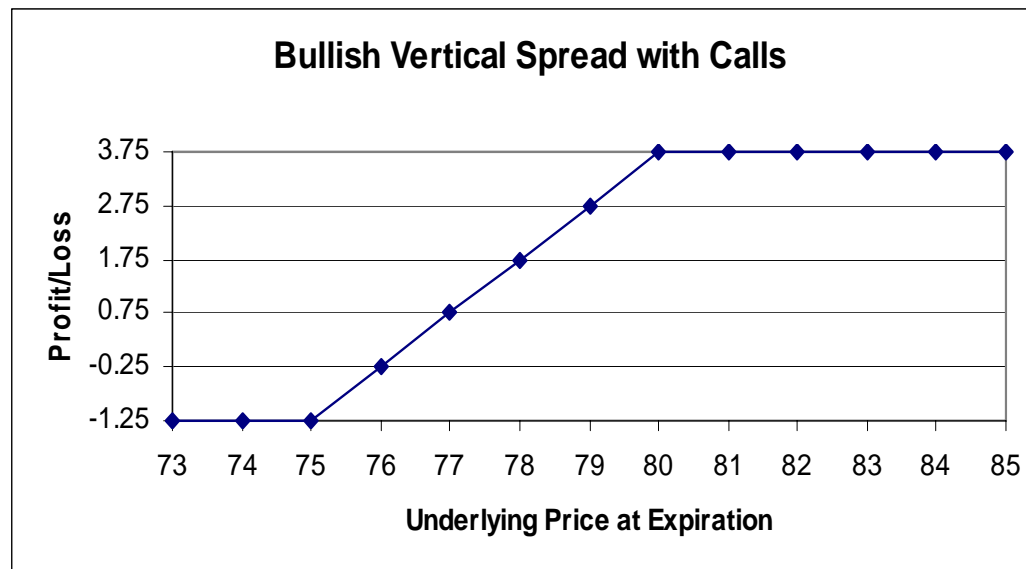
S_T	C(T) Offset 75 Call	C(T) offset 80 Call	CF(T)	Total Profit CF(0)+CF(T)
73	0.00	0.00	0.00	(1.25)
74	0.00	0.00	0.00	(1.25)
75	0.00	0.00	0.00	(1.25)
76	1.00	0.00	1.00	(0.25)
77	2.00	0.00	2.00	0.75
78	3.00	0.00	3.00	1.75
79	4.00	0.00	4.00	2.75
80	5.00	0.00	5.00	3.75
81	6.00	(1.00)	5.00	3.75
82	7.00	(2.00)	5.00	3.75
83	8.00	(3.00)	5.00	3.75
84	9.00	(4.00)	5.00	3.75
85	10.00	(5.00)	5.00	3.75



Example: Bullish Vertical Spread with Calls, III.

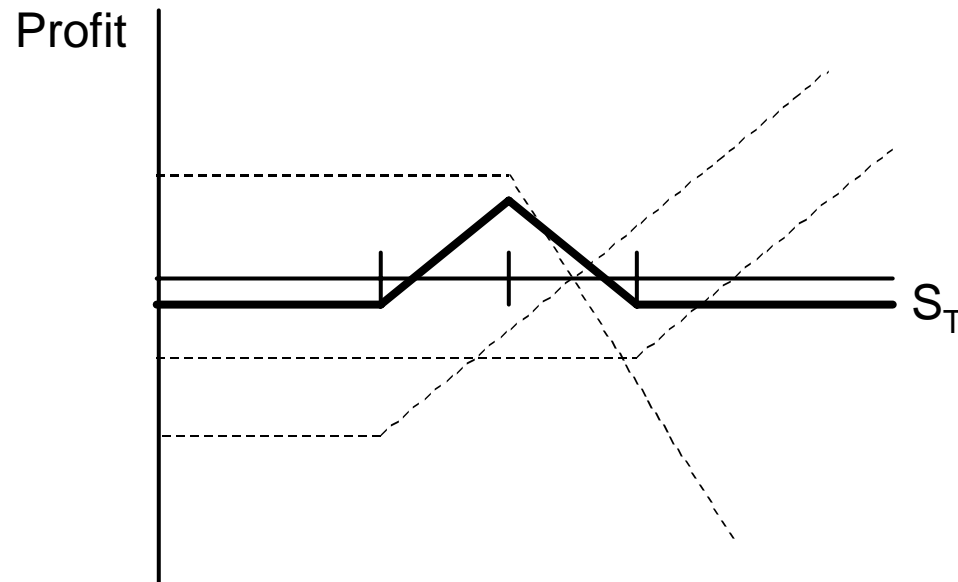
Then, one can plot the underlying price at expiration against the position profit or loss (note that the kinks are at the strike prices, 75 and 80):

(Obviously, one could plot each elementary position as well.)



Butterfly Spread Using Calls

- This is a Long Call Butterfly: With equally spaced strikes:



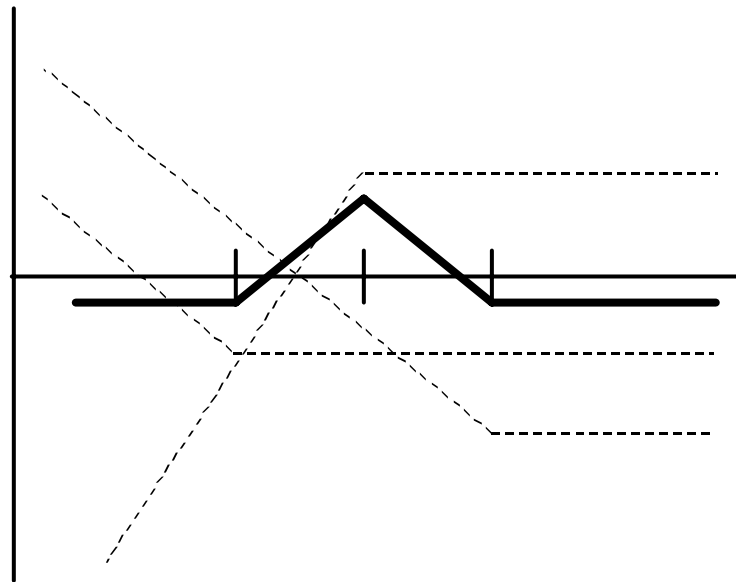
Long Butterfly Using Calls

**Long 1 with lowest strike;
Short 2 with middle strike;
Long 1 with highest strike**



Butterfly Spread Using Puts

- This is a Long Put Butterfly: With equally spaced strikes:



Long 1 with lowest strike;
Short 2 with middle strike;
Long 1 with highest strike

Long Butterfly Using Puts

What do you think a **written** butterfly would look like?



Other Spreads, I.

- Calendar Spreads:
 - Use the same strike, but with two different expiration dates.
 - Can use either calls or puts.
 - The resulting payoff is curved. This is because one option is still ‘alive’ at the expiration date of the other.
- Ratio Spreads (pg. 430)
 - Can use either calls or puts.
 - Same expiration, but with two different strikes.
 - **However**, unlike other spreads, the number of options held in each position is not the same. For example, a one could buy 3 puts with a strike of 30, and sell one put with a strike of 35.



Other Spreads, II.

- Condor Spread.
 - Uses four, equally spaced strikes.
 - For a long condor spread: Long 1 at the lowest and 1 at the highest strike; short 1 at both intervening strikes.
 - The resulting payoff resembles a butterfly spread, but with a ‘flat spot’ between the middle two strikes. (The payoff for a long butterfly resembles a ‘witches’ hat; the payoff for a long condor resembles a ‘stovepipe’ hat.)



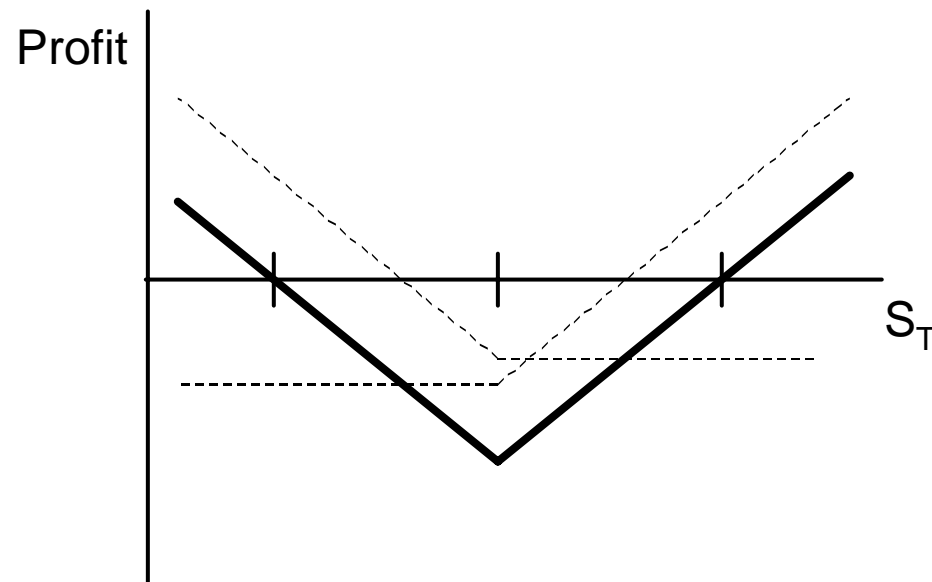
Other Spreads, III.

- Box 'Spread' (Really, these are **combinations**)
 - Use two equally spaced strikes, K_1 and K_2 , where $K_1 < K_2$.
 - **Long Box: Long a call with strike K_1 ; Long a put with strike K_2 .
Short a call with strike K_2 ; Short a put with strike K_1 .**
 - A **Long Box** costs money today, but always has a value of $K_2 - K_1$ at expiration. Therefore, a long box resembles riskless **lending, i.e., long T-bill.**
 - A **Short Box** is formed by reversing all the positions in a long box. As a result, a short box generates a cash inflow today, but has a value of $-(K_2 - K_1)$ at expiration. Therefore, a short box resembles riskless **borrowing, i.e., short T-bill.**



Combinations, I.

- A Long **Straddle** is formed by a long call and a long put:
 - Both have the same strike and expiration date.
 - What is the **worst** possible value for the underlying at expiration?
 - In a Short Straddle, one sells the call and sells the put.

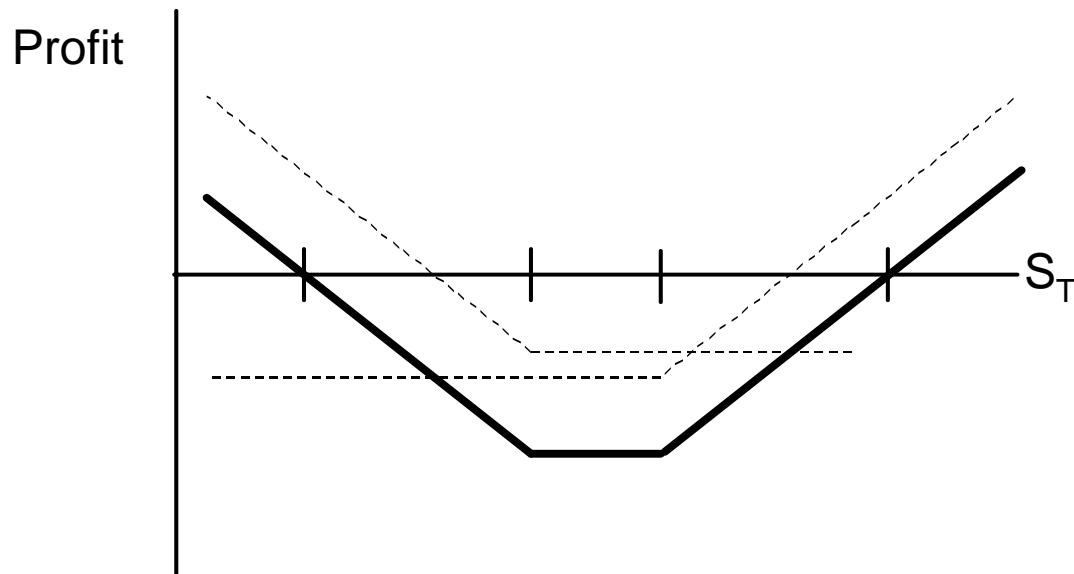


Long Straddle Using a Call and a Put



Combinations, II.

- A Long **Strangle** is formed by a long call and a long put:
 - Both have the same expiration date.
 - But, the call and put have different strike prices.
 - In a **Short Strangle**, one sells the call and sells the put.
(what does it look like?)

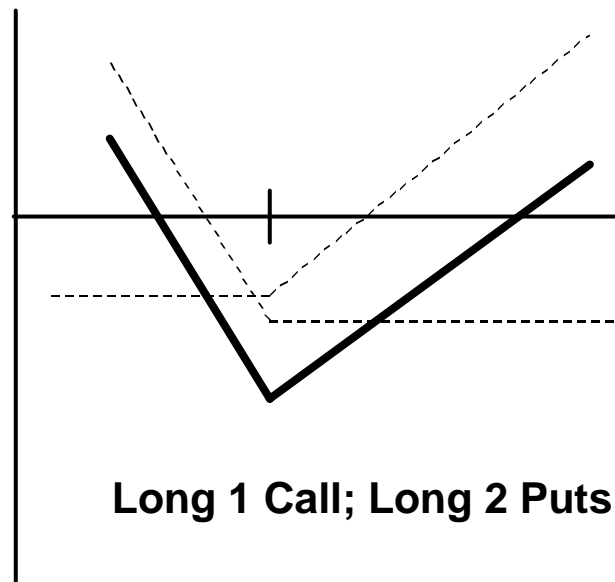


Long Strangle Using a Call and a Put



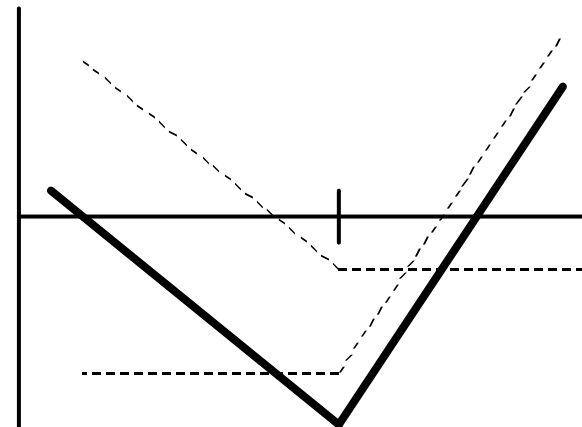
Combinations, III. Strips and Straps

- Strips and straps are formed by using a different number of calls and puts. However, all the options share
 - The same strike price.
 - The same expiration date.



Long 1 Call; Long 2 Puts

[A] Long Strip



Long 1 Put; Long 2 Calls

[B] Long Strap

What are the slopes of these lines?



Example: Long 85 Straddle

- You see the following option data and decide to purchase an 85 call and an 85 put.

Strike	Call	Put
75	11.50	0.75
80	7.00	1.38
85	4.25	3.25
90	2.25	6.13
95	0.81	8.88

- Using the steps to build a profit table, you construct the following table.



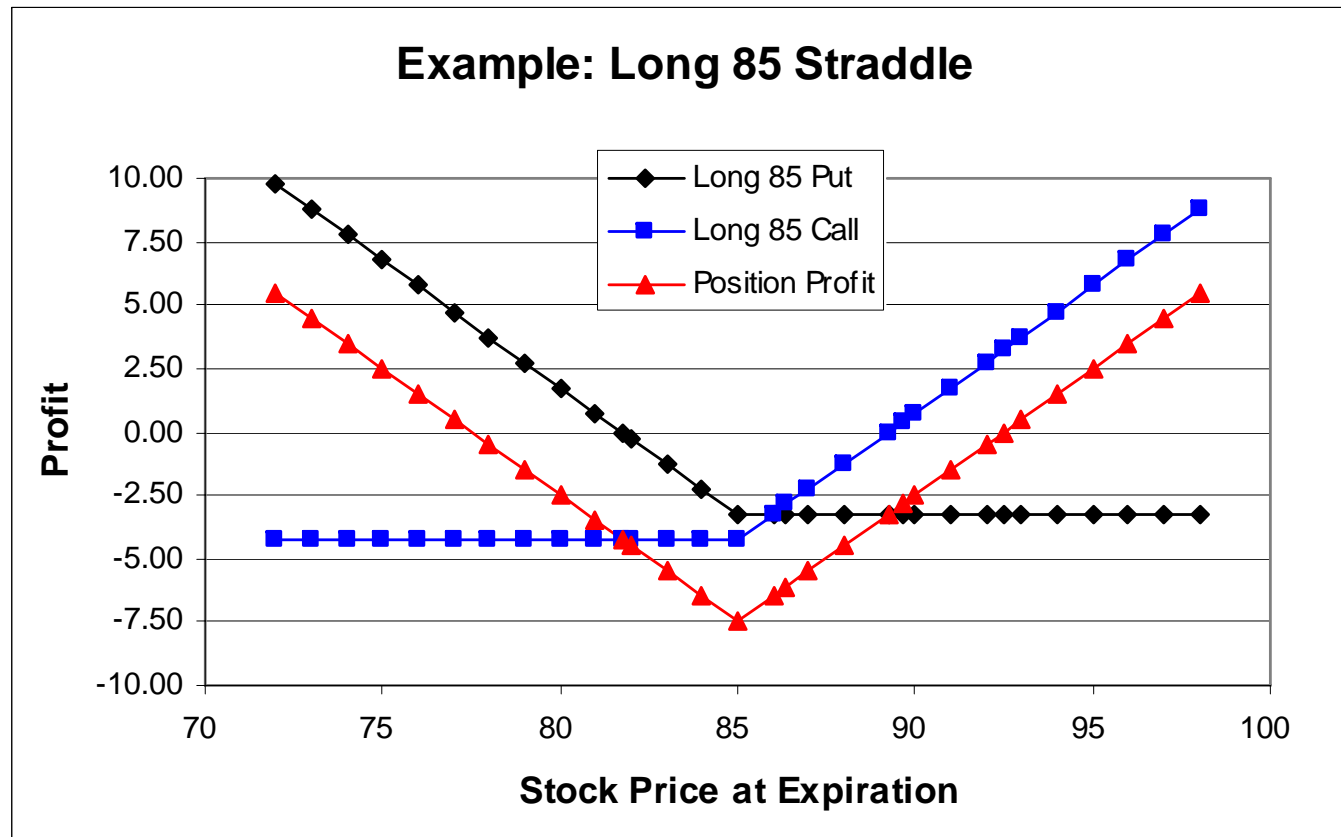
Long 85 Straddle, II.

		Stock Price at Expiration	Offset P(T) 85 Put	Offset C(T) 85 Call	CF(T)	CF(0)	Portfolio Profit
Time 0		75.00	10.00	0.00	10.00	(7.50)	2.50
Buy C (K=85)	-4.25	76.00	9.00	0.00	9.00	(7.50)	1.50
		77.00	8.00	0.00	8.00	(7.50)	0.50
		78.00	7.00	0.00	7.00	(7.50)	(0.50)
Buy P (K=85)	-3.25	79.00	6.00	0.00	6.00	(7.50)	(1.50)
		80.00	5.00	0.00	5.00	(7.50)	(2.50)
		81.00	4.00	0.00	4.00	(7.50)	(3.50)
CF(0)	-7.50	81.75	3.25	0.00	3.25	(7.50)	(4.25)
		82.00	3.00	0.00	3.00	(7.50)	(4.50)
		83.00	2.00	0.00	2.00	(7.50)	(5.50)
		84.00	1.00	0.00	1.00	(7.50)	(6.50)
		85.00	0.00	0.00	0.00	(7.50)	(7.50)
		86.00	0.00	1.00	1.00	(7.50)	(6.50)
		86.38	0.00	1.38	1.38	(7.50)	(6.12)
		87.00	0.00	2.00	2.00	(7.50)	(5.50)
		88.00	0.00	3.00	3.00	(7.50)	(4.50)
		89.25	0.00	4.25	4.25	(7.50)	(3.25)
		89.63	0.00	4.63	4.63	(7.50)	(2.87)
		90.00	0.00	5.00	5.00	(7.50)	(2.50)
		91.00	0.00	6.00	6.00	(7.50)	(1.50)
		92.00	0.00	7.00	7.00	(7.50)	(0.50)
		92.50	0.00	7.50	7.50	(7.50)	0.00
		93.00	0.00	8.00	8.00	(7.50)	0.50
		94.00	0.00	9.00	9.00	(7.50)	1.50
		95.00	0.00	10.00	10.00	(7.50)	2.50



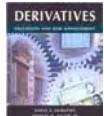
Long 85 Straddle, III.

Then, one can plot the profit data:



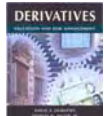
Profit Diagrams for Positions Offset Prior to the Expiration Day

- Any strategy can be offset prior to expiration.
- To prepare a profit diagram (as a function of the price of the underlying asset on a given day prior to T), you must estimate the value of the options. For this, you need an option pricing model. You also have to guess what implied volatility (σ) will exist in the option prices on that day.
- See pg. 434 for diagrams depicting how a bullish money spread and a long straddle evolve over time.



Expectations of Students

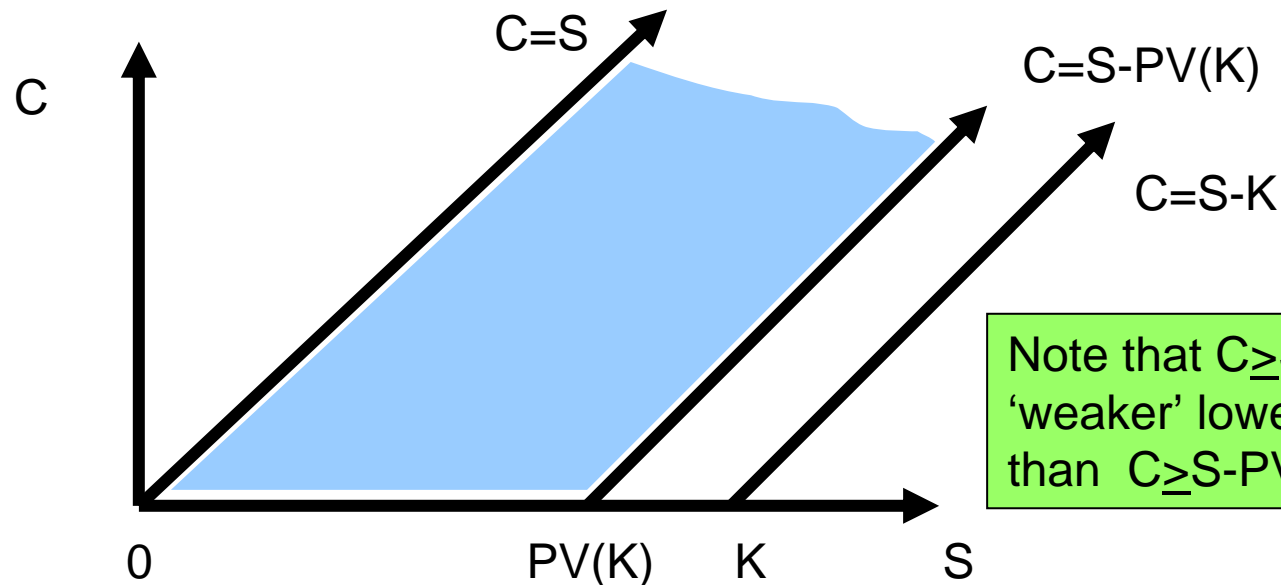
- You should know what the following strategies are, and what their profit diagrams look like:
 - Long stock, short stock
 - Long call, short call, long put, short put
 - Covered call, protective put
 - Bullish money spread and bearish money spread
 - Long and short straddle and strangle
- BUT...I can give you any melange of elementary positions, and you should be able to prepare a profit table. See, for example, problem 15.10.



Chapter 16

Arbitrage Restrictions on Option Prices

- Calls
 - The **upper bound** is $C \leq S$
 - If there are no dividend payouts, the **lower bound** is $C \geq \max [0, S - K(1+r)^{-T}]$; i.e., $C \geq \max [0, S - PV(K)]$



Proof that $C \geq \text{Max}[0, S - \text{PV}(K)]$

- Obviously, $C \geq 0$
- **Proof that $C \geq S - \text{PV}(K)$**
 - What if $C < S - \text{PV}(K)$?
 - Then $C - S + \text{PV}(K) < 0$
 - Then $-C + S - \text{PV}(K) > 0$ permits arbitrage, because cash is received today, and there are no cash outflows at expiration.

		At Expiration	
		$S_T > K$	$S_T < K$
– Today:			
• Buy call	-C	$+(S_T - K)$	0
• Sell stock	+S	$- S_T$	$- S_T$
• Lend	<u>-PV(K)</u>	<u>+ K</u>	<u>+ K</u>
	>0	0	$-S_T + K > 0$



Proof that $C \geq \text{Max}[0, S - \text{PV}(K)]$: A Numerical Example

- Let: $C = 4$, $S = 44$, $K = 40$, $r = 6\%$, $T = 1 \text{ mo.}$, $\text{PV}(K) = 39.80$
- What if $4 < 44 - 39.80 = 4.20$?
- Then $4 - 44 + 39.80 = -0.20 < 0$
- Then $-4 + 44 - 39.80 = +0.20 > 0$ permits an arbitrage

		At Expiration	
		<u>$S_T = 42$</u>	<u>$S_T = 37$</u>
- Today:			
• Buy call	-4	+2	0
• Sell stock	+44	- 42	- 37
• <u>Lend</u>	<u>-39.80</u>	<u>+ 40</u>	<u>+ 40</u>
	+0.20	0	+3



For **American** Calls Only

- The weaker lower bound is $C > S - K$
- Violation of this bound permits “instant arbitrage”.
- Proof:
 - What if $C < S - K$? (i.e., the call is selling for less than its intrinsic value; e.g., $S=44$, $K = 40$ and $C = 3.90$).

– Then,

Buy call	-C	-3.90
Exercise call	-K	-40 (thereby acquiring the stock)
Sell stock	<u>+S</u>	<u>+44</u>
	>0	+0.10



Theoretical Implications of these Bounds

- Because the price of an in-the-money call must exceed $S - PV(K)$, in the money calls on non-dividend paying stocks will **always** have some time value before expiration.
- An American call on a non-dividend paying stock will **never** be exercised before expiration. **Why exercise for $S - K$, when you can sell it for $S - PV(K)$ or more?**
- The value of an **American** call on a non-dividend paying stock is the same as the value of a **European** call on the same stock, all else equal.



Lower bound for **European** calls on **Dividend Paying Stocks**

- $C \geq \max [0, S - \bar{D}(1+r)^{-t_1} - K(1+r)^{-T}]$
- \bar{D} is the highest dividend possible paid at time $t_1 < T$ (it is also assumed that the ex-dividend date and the dividend payment date are the same).
- This means that deep in the money **European** calls on dividend paying stocks **can** sell for less than their intrinsic value.
 - Example: if $S=60$, $K=20$, $T=1$ year, $r=10\%$, $PV(K)=18.18$, $PV(D) = 3$, then the lower bound is $C \geq 60 - 3 - 18.18 = 38.82$. But intrinsic value is 40. Thus, the **European** call can sell for less than its intrinsic value, with no possible arbitrage.



Lower bound for **American** calls on **Dividend Paying Stocks**

- Proposition IV:

$$C \geq \text{MAX} \begin{cases} \text{(a)} & 0 \\ \text{(b)} & S - K(1 + r)^{-t_1} \\ \text{(c)} & S - \overline{D1} (1 + r)^{-t_1} - K(1 + r)^{-t_2} \\ \text{(d)} & S - \overline{D1} (1 + r)^{-t_1} - \overline{D2} (1 + r)^{-t_2} - K(1 + r)^{-T} \end{cases}$$

- Because of IVb, we can state that :
 - ***An in the money **American** call on a dividend paying stock will always have some time value, except (possibly) on the day before it trades ex-dividend, and (always) on its expiration day.***
 - ***An in the money **American** call will never be exercised early, except on the day before it trades ex-dividend (if IVb is binding, and t_1 is one day (1/365 year)).***



When will an **American** call be Exercised Early? The Logic

- Exercising an option early destroys its time value, so you will only want to exercise early if the option has no time value.
- Exercising early also destroys the downside protection provided by a call (i.e., if $S_T < K$), and requires spending $\$K$ earlier (thereby losing interest that can be earned on $\$K$).
- So, the dividend that will be paid “tomorrow” must be sufficiently great to compensate you for these “costs” of early exercise.



When will an **American** Call be Exercised Early? The Algebra

- Suppose that today, the day before a stock trades ex-dividend, a call is selling for its intrinsic value:

$$C(\text{with-div}) = S(\text{with-div}) - K$$

- Tomorrow, on the ex-dividend day, the call's lower bound is:

$$C = S(\text{ex-div}) - PV(K)$$

- Assume that $E\{C(\text{ex-div})\} = S(\text{ex-div}) - PV(K) + E(\tilde{z})$

- You will want to exercise the call today if:

$$C(\text{with-div}) > E\{C(\text{ex-div})\}$$

$$S(\text{with-div}) - K > E\{S(\text{ex-div})\} - PV(K) + E(\tilde{z})$$

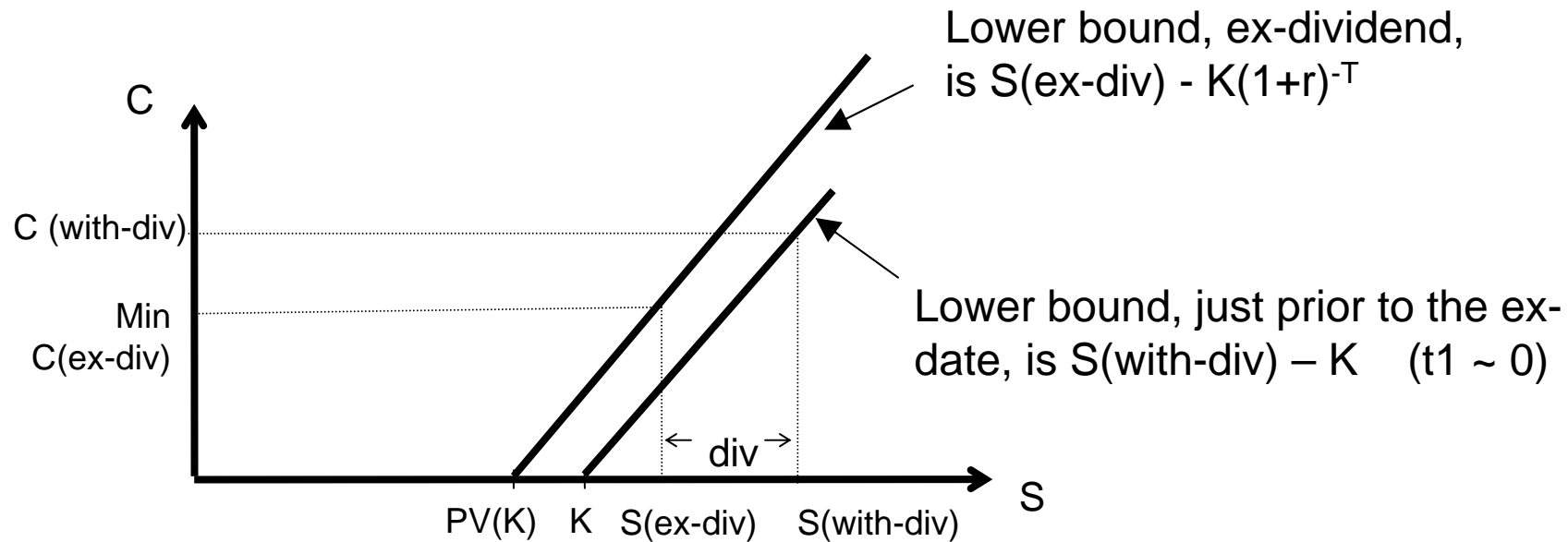
Assume $E(z) = 0$, and since $E\{S(\text{ex-div})\} = S(\text{with-div}) - \text{div}$,

$$\mathbf{Div > K - PV(K)}$$

- I.e., you will want to exercise early if the dividend exceeds the interest that can be earned by investing $PV(K)$ until the expiration day.



When will an **American** call be exercised early? The Diagram



It is possible that the American call will fall in value from $C(\text{with-div})$ to as low as $C(\text{ex-div})$, on the ex-dividend day. American call owners will exercise early, rather than watch the call decline in value.



Arbitrage Restrictions on Put Prices

- Assume the stock pays no dividends.
- Upper bounds are
 - **American**: $P \leq K$
 - **European**: $P \leq K(1+r)^{-T}$
- Lower bounds are
 - **American**: $P \geq \max(0, K-S)$
 - **European**: $P \geq \max(0, K(1+r)^{-T} - S)$
- Thus, an **American** put cannot sell for less than its intrinsic value, but a **European** put can!



Proof of the **American** Put Lower Bound

- What if $P < K - S$?
Then, $P - K + S < 0$
Or, $-P + K - S > 0$

Today:

Buy put	- P
Buy stock	- S
Exercise put	<u>+ K</u>
	> 0

- Numerical Example.
- What if $P = 2.70$ when $K = 40$ and $S = 37$? ($2.70 < 40 - 37$)

Today:

Buy put	-2.70
Buy stock	- 37
Exercise put	<u>+ 40</u>
	+ 0.30



Proof of the **European** Put Lower Bound

What if: $P < K(1+r)^{-T} - S$?

Then, $P - K(1+r)^{-T} + S < 0$

Or, $-P + K(1+r)^{-T} - S > 0$

At expiration:

<u>Today</u>		<u>$S_T > K$</u>	<u>$S_T < K$</u>
Buy put	-P	0	+(K - S_T)
Borrow	+ $K(1+r)^{-T}$	-K	-K
Buy stock	<u>-S</u>	<u>+S_T</u>	<u>+S_T</u>
	>0	<u>≥0</u>	0

So, if $P < K(1+r)^{-T} - S$, an arbitrage is possible, because the trader can receive a cash in-flow today, and not have to pay money in the future (in fact, in some cases, the trader receives money in the future, too).



Put Pricing Bounds: The Diagram

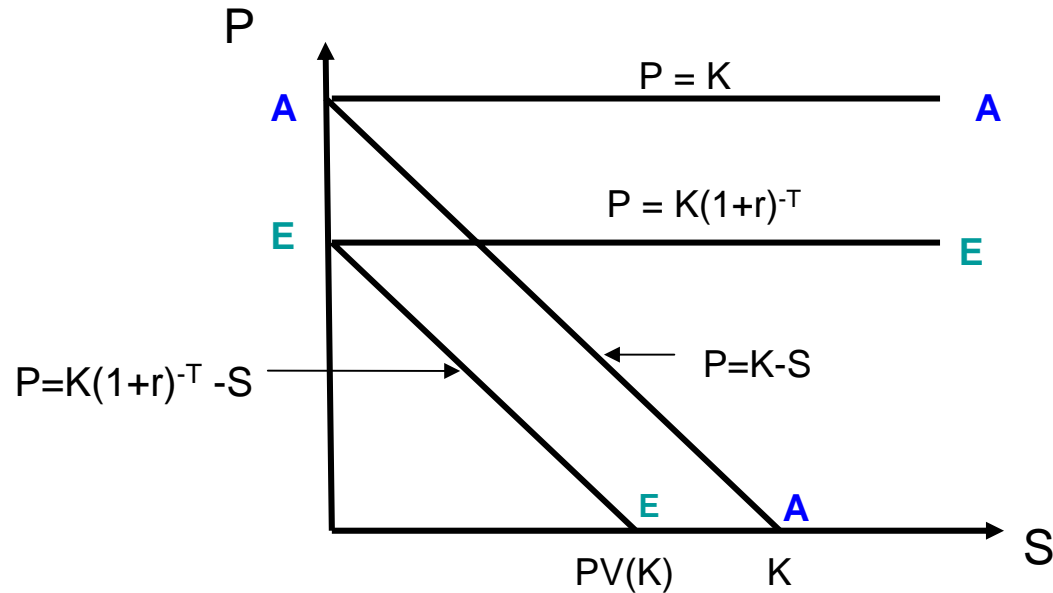


Figure 16-5

Pricing Boundaries for Puts on Non-Dividend Paying Stocks

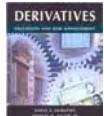
AAA shows the **American** put boundaries.

EEE shows the **European** put boundaries



Early Exercise of **American** Puts

- Once an **American** put is sufficiently in-the-money, it will sell for its intrinsic value, and it should then be exercised early.
- Exercising early will get you \$K today, rather than at expiration, and you can immediately invest that money to earn interest.



Put-Call Parity

- For **European** options on non-dividend paying stocks, put-call parity is:

$$C - P = S - PV(K)$$

- For **European** options on stocks paying known dividends, put-call parity is:

$$C - P = S - PV(\text{Divs}) - PV(K)$$

- For **European** options on stocks paying unknown dividends, put-call parity is:

$$S - PV_L(\text{Divs}_L) - PV(K) \geq C - P \geq S - PV^H(\text{Divs}^H) - PV(K)$$



Proof of the Basic Put-Call Parity Proposition: A Conversion

What if: $C - P > S - K(1+r)^{-T}$?

Then: $C - P - S + K(1+r)^{-T} > 0$

<u>Today:</u>		<u>At Expiration:</u>	
		<u>$S_T < K$</u>	<u>$S_T > K$</u>
Sell call	+C	0	$-(S_T - K)$
Buy put	-P	$+(K - S_T)$	0
Buy stock	-S	$+S_T$	$+S_T$
Borrow	<u>$+K(1+r)^{-T}$</u>	<u>-K</u>	<u>-K</u>
	>0	0	0

Therefore, if $C - P > S - K(1+r)^{-T}$, an arbitrage is possible, because the trader receives a cash inflow today, but does not have a cash out-flow in the future.



Exploiting a Violation of Put-Call Parity: An Example of a “Reverse Conversion”

Suppose $C = 4.50$, $P = 2.50$, $S = 42$, $K = 40$, $r = 6\%$, $T = 3$ mos., $PV(K) = 39.41$.

$$C - P = 4.5 - 2.5 = 2 < S - PV(K) = 42 - 39.41 = 2.59.$$

<u>Today:</u>		<u>At Expiration:</u>	
		<u>$S_T=37$</u>	<u>$S_T=44$</u>
Buy call	-4.50	0	+4
Sell put	+2.50	-3	0
Sell stock	+42.00	-37	-44
Lend	<u>-39.41</u>	<u>+40</u>	<u>+40</u>
	+0.59	0	0



Some Theoretical Implications of Put-Call Parity

- Rearrange, the basic put-call parity proposition to be $-C = -S + PV(K) - P$. This says that buying a call is like borrowing to buy stock; i.e., it is like buying stock on margin. But in addition, the call owner also owns a put, providing downside protection.
- If $r > 0$, an at the money call is worth more than an at the money put with the same K and T .
- Given S , r , and T , then $C - P$ is known, regardless of the bullishness or bearishness that may pervade the market.
- You can replicate the payoff from any position with the other three securities (e.g. buying a put = selling stock, lending, and buying a call).



Put-Call Parity & Synthetic Positions

- $S = C - P + PV(K)$
- $-S = -C + P - PV(K)$
- $-P = S - C - PV(K)$
- $P = -S + C + PV(K)$
- $C = S + P - PV(K)$
- $-C = -S - P + PV(K)$
- $PV(K) = S - C + P$
- $-PV(K) = -S + C - P$

Sell stock short = write call, buy put, & borrow

Buy stock = buy call, write put & lend

Buy put = sell stock short, buy call, & lend

Write put = buy stock, write call, & borrow

Write call = sell stock short, write put, & lend

Buy call = buy stock, buy put, & borrow

Riskless borrowing = sell stock short,
buy call, & write put

Riskless lending = buy stock, write call,
& buy put



Put-Call Parity: What About **American** Options?

- Either a put or a call is always written when arbitraging a put-call violation.
- Therefore, an arbitrageur must guarantee that arbitrage profits will be realized even if:
 - the written **American** call is exercised early (if the stock pays a dividend), or
 - the written **American** put is exercised early.



American Put-Call Parity

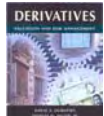
- No dividends (only the written put might be exercised early):

$$S - K(1+r)^{-T} \geq C - P \geq S - K$$

- Stocks that pay dividends (the written call or the written put might be exercised early):

$$S - PV(K) \geq C - P \geq S - PV(D^H) - K$$

- This last inequality can be quite wide.



American Put-Call Parity: An Example

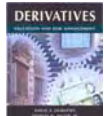
Using GM at 3pm on February 15, 2000.

- $S=74.50$, $K=75$, $C=9$, $P=8.75$, $r=6\%$ per year, $T=221$ days = 0.6055 year, $K(1+r)^{-T} = 75(1.06)^{-0.6055} = 72.40$.
- The last ex-dividend date was on 2/7/2000, and the dividend amount was \$0.50 per share. Thus, assume $D_1^H=0.55$, $t_1=90$ days=0.2466 year, $PV(D_1^H) = 0.5422$, and $D_2^H=0.60$, $t_2=182$ days=0.4986 year, $PV(D_2^H) = 0.5828$.
- $S - PV(K) \geq C - P \geq S - PV(D^H) - K$
- $74.5 - 72.4 \geq 9 - 8.75 \geq 74.5 - 0.5422 - 0.5828 - 75$
- $2.1 \geq 0.25 \geq -1.625$
- Thus, there are no arbitrage opportunities. However, notice the width of the no-arbitrage range, $C - P$.



Some Extra Slides on this Material

- Note: In some chapters, we try to include some extra slides in an effort to allow for a deeper (or different) treatment of the material in the chapter.
- If you have created some slides that you would like to share with the community of educators that use our book, please send them to us!



We are now going to price options before expiration. But before we do:

- The ‘Hockey Sticks’ have helped you learn an important concept: **Intrinsic Value**.
 - **For calls: $\text{MAX}(S - K, 0)$**
(You read this as: “The maximum of: the stock price minus the strike price OR zero.”)
 - **For puts: $\text{MAX}(K - S, 0)$**
(You read this as: “The maximum of: the strike price minus the stock price OR zero.”)



Intrinsic Value, I.

- It is **really** vital that you remember:

Intrinsic Value can be calculated whether an option is ‘dead or alive.’

- That is, you can always calculate the intrinsic value of an option.



Intrinsic Value, II.

- At expiration, the value of an option is just its **intrinsic value**.
- Before expiration, the value of an option is the sum of **intrinsic value** and time value.
- Therefore, before an option expires, you can *always* calculate its **intrinsic value**.



Intrinsic Value, III.

- **Example 1. Suppose a call option exists with 21 days to expiration. Suppose this call is selling for \$1.68. The underlying asset price is \$41.12.**
 - Calculate the intrinsic value of a call with a strike price of 40. What is the time value?
 - Calculate the intrinsic value of a call with a strike price of 45. What is the time value?



Intrinsic Value, IV.

- **Example 2. Suppose a put option exists with 21 days to expiration. Suppose this put is selling for \$5.68. The underlying asset price is \$41.12.**
 - Calculate the intrinsic value of a put with a strike price of 40. What is the time value?
 - Calculate the intrinsic value of a put with a strike price of 45. What is the time value?



Intrinsic Value, IV.

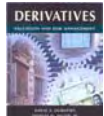
- **Example 3. Now, suppose there are 0 days to expiration.**
- **Recalculate the intrinsic values for the two calls and the two puts. (What is the time value here?)**



Pricing Options Before Expiration, I. Put-Call Parity

In what follows, the signs are very important.

- Long Position (+)
- Short Position (-)



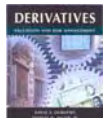
- If we are long a stock and long a put, then we have replicated a long call position.

$$S + P = C \text{ (do they cost the same?)}$$

- If we are long a stock and short a call, then we have replicated a short put position.

$$S - C = -P$$

- The purple equal sign (=) means that the two sides are not equal in cost, but are equal in replicating payoffs.



- If we are long a call and short a put, then we have replicated a long stock position.

$$C - P = S \text{ (cost the same?)}$$

- If we are long a put and short a call, then we have replicated a short stock position.

$$P - C = -S$$

- By now, you may have noticed that the above equations are all versions of the same equation. This is not a fluke.



Actual Long Call and Synthetic Long Call Position

Suppose, At Expiration:

	<u>S < K</u>	<u>S = K</u>	<u>S > K</u>
Actual Long Call:	0	0	S - K
Synthetic Long Call:			
Long put	K - S	0	0
Long stock	S	S	S
Borrow	<u>-K</u>	<u>-K</u>	<u>-K</u>
	0	0	S - K



- Reviewing some synthetic creations:
- A short call and a long put creates a synthetic short stock position. (Note: same strike price required.)
- To hedge this position, borrow money and buy stock.



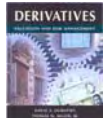
Therefore, at expiration:

	<u>S < K</u>	<u>S = K</u>	<u>S > K</u>
Short Call:	0	0	-(S-K)
Long Put:	K-S	0	0
Long stock:	<u>S</u>	<u>S</u>	<u>S</u>
Borrow:	<u>-K</u>	<u>-K</u>	<u>-K</u>

What does it mean?



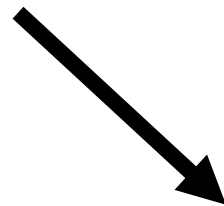
- Today, you borrow the an amount necessary to buy the put, buy the stock, and sell the call, I.e., $P+S-C$.
- At expiration, you have an asset worth K .
- Note that this asset is riskless.
- Therefore, you can finance this asset today by borrowing the present value of K .
- This yields a very important equation:



The Put-Call Parity Condition:

$$P + S - C = \frac{K}{(1+r)} \quad P + S - C - \frac{K}{(1+r)} = 0$$

OR,



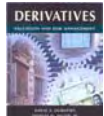
$$P + S - \frac{K}{(1+r)} = C$$



Sometimes, this is called the “no-arbitrage” condition

$$(S + P - C)(1+r) = K$$

Now, if you are looking for a really useful tattoo.....



PCP allows us to price options before expiration, but.....

- We need a call price to price a put using put-call parity
- We need a put price to price a call using put-call parity

Nonetheless, PCP has some handy uses:



Read the signs. They Matter.

To create a synthetic long put:

$$P = C - S + K/(1+r)$$

- Buy the call,
- Sell the stock,
- Buy T-bills (invest at the risk-free rate)



Read the signs. They Matter.

To create a synthetic short put:

$$-P = -C + S - K/(1+r)$$

- Sell the call,
- Buy the stock,
- Sell T-bills, (borrow at the risk-free rate)



Read the signs (maybe from your new tattoo!) They Matter.

To create a synthetic long call:

$$C = P + S - K/(1 + r)$$

- Buy the put,
- Buy the stock,
- Sell T-bills (borrow at the risk-free rate)



You know what to do....

To create a synthetic short call:

$$-C = -P - S + K/(1 + r)$$

- Sell the put,
- Sell the stock,
- Buy T-bills (invest at the risk-free rate)



Arbitrage Example

Suppose:

$$S = 40$$

$$K = 40$$

$$C = \$3$$

$$P = \$2$$

$$r = 6\%/year$$

$$T = 3 \text{ months}$$

Is there an arbitrage opportunity? Hint, yes.



Today:

Buy put: - \$2

Sell call: + \$3

Buy stock: + \$40

Borrow: + \$39

At expiration:

We receive: $K = \$40$

We repay: $(\$39)(1.015) = \39.59

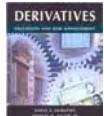
Difference of \$0.41



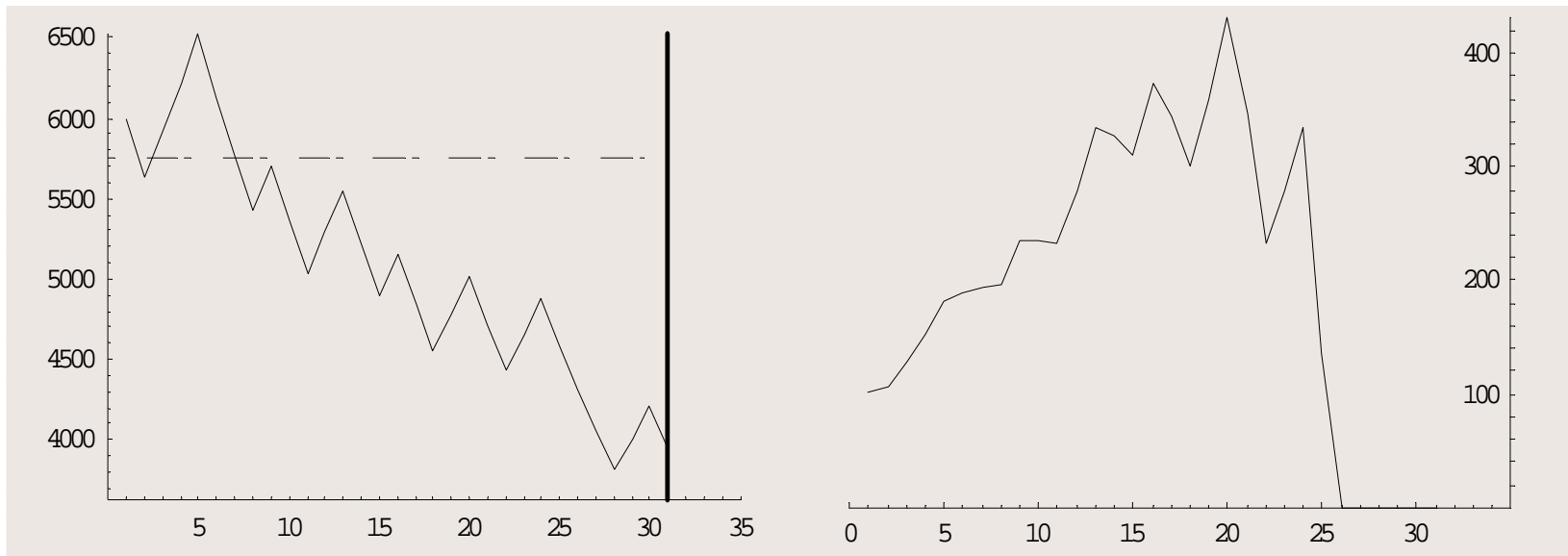
Chapter 17

The Binomial Option Pricing Model (BOPM)

- We begin with a single period.
- Then, we stitch single periods together to form the Multi-Period Binomial Option Pricing Model.
- The Multi-Period Binomial Option Pricing Model is extremely flexible, hence valuable; it can value American options (which can be exercised early), and most, if not all, exotic options.

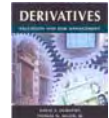


Stock and Call Price



Stock Price

Call Price



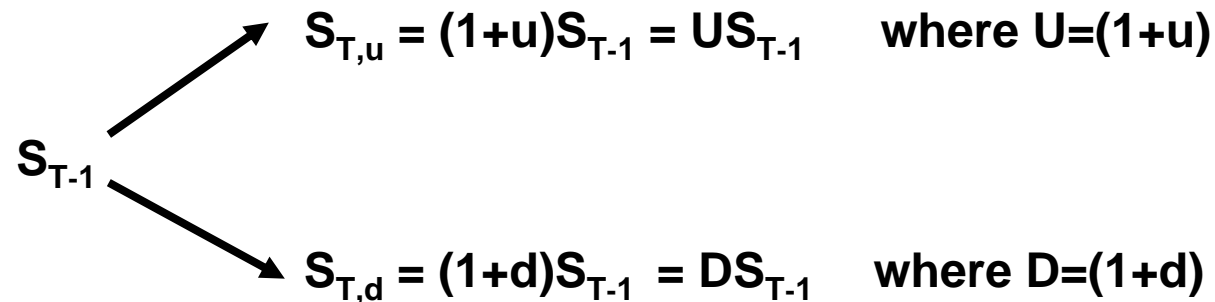
Assumptions of the BOPM

- There are two (and only two) possible prices for the underlying asset on the next date. The underlying price will either:
 - Increase by a factor of $u\%$ (an uptick)
 - Decrease by a factor of $d\%$ (a downtick)
- The uncertainty is that we do not know which of the two prices will be realized.
- No dividends.
- The one-period interest rate, r , is constant over the life of the option ($r\%$ per period).
- Markets are perfect (no commissions, bid-ask spreads, taxes, price pressure, etc.)

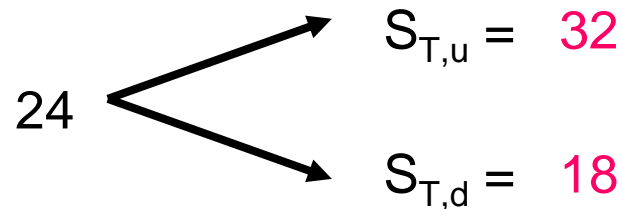


The Stock Pricing 'Process'

Time T is the expiration day of a call option. Time T-1 is one period prior to expiration.



Suppose that $S_{T-1} = 24$, $u = 1/3$ (33.33%) and $d = -1/4$ (-25%) ($U=4/3$, $D=3/4$). What are $S_{T,u}$ and $S_{T,d}$?



The Option Pricing Process

$$C_{T-1} \begin{cases} \rightarrow C_{T,u} = \max(0, S_{T,u} - K) = \max(0, (1+u)S_{T-1} - K) \\ \rightarrow C_{T,d} = \max(0, S_{T,d} - K) = \max(0, (1+d)S_{T-1} - K) \end{cases}$$

Suppose that $K = 25$. What are $C_{T,u}$ and $C_{T,d}$?

$$C_{T-1} \begin{cases} \rightarrow C_{T,u} = 7 \\ \rightarrow C_{T,d} = 0 \end{cases}$$



The Equivalent Portfolio (Synthetic)

Buy Δ shares of stock and invest B in riskless bonds.

$$\Delta S_{T-1} + B \begin{cases} \nearrow \Delta(1+u)S_{T-1} + (1+r)B = \Delta S_{T,u} + (1+r)B \\ \searrow \Delta(1+d)S_{T-1} + (1+r)B = \Delta S_{T,d} + (1+r)B \end{cases}$$

NB: Δ is **not a “change” in S It defines the # of shares to buy. For a call, $0 \leq \Delta \leq 1$**

Set the payoffs of the equivalent portfolio equal to $C_{T,u}$ and $C_{T,d}$, respectively.

$$\begin{aligned} \Delta(1+u)S_{T-1} + (1+r)B &= C_{T,u} \\ \Delta(1+d)S_{T-1} + (1+r)B &= C_{T,d} \end{aligned} \quad \leftarrow \text{These are two equations with two unknowns: } \Delta \text{ and } B$$

What are the two equations in the numerical example with $S_{T-1} = 24$, $u = 1/3$, $d = -1/4$, $r = 1/8$ (12.5%), and $K = 25$?



A Key Point

- If two assets offer the same payoffs at time T , then they must be priced the same at time $T-1$.
- Here, we have set the problem up so that the equivalent portfolio offers the same payoffs as the call.
- Hence the call's value at time $T-1$ must equal the \$ amount invested in the equivalent portfolio.

$$C_{T-1} = \Delta S_{T-1} + B$$



So, in the Numerical Example....

$S_{T-1} = 24$, $u = 1/3$, $S_{T,u} = 32$, $d = -1/4$, $S_{T,d} = 18$, $r = 1/8$ (12.5%), $K = 25$,
 $C_{T,u} = 7$ and $C_{T,d} = 0$.

What are the values of Δ , B , and C_{T-1} ?

$$32 \Delta + (1 + 1/8)B = 7$$

$$18 \Delta + (1 + 1/8)B = 0$$

Solving

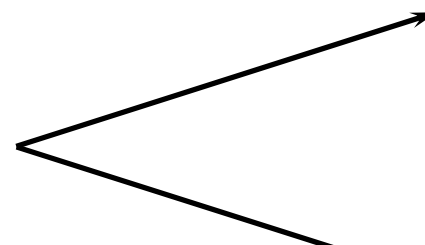
$$14 \Delta = 7 \quad \text{or} \quad \Delta = \frac{1}{2} \text{ (buy half unit of stock)}$$

$$9 + (9/8)B = 0 \quad \text{or} \quad B = -8 \text{ (borrow 8)}$$



The Synthetic Option

- The equivalent portfolio is
long 1/2 shares
borrow 8


$$(32/2) - (9/8)8 = 16 - 9 = 7$$
$$(18/2) - (9/8)8 = 9 - 9 = 0$$

$$\text{Cost} = (24/2) - 8 = 4.$$

In the absence of arbitrage opportunities therefore:

$$\text{call price} = 4$$

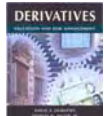
What if $C_{T-1} = 5$?

What if $C_{T-1} = 3$?



The call option price depends on

- Current stock price, S_{T-1}
- The up and down factors, u and d (volatility).
- The strike price, K .
- The interest rate, r .



It doesn't depend on

- Irrelevance of Stock's Expected Return
 - C does not depend on *probability* of up or down movements in the stock price
- Irrelevance of investors' risk aversion
 - no assumptions were made about risk preferences in deriving C
- Irrelevance of the market portfolio
 - C depends only on the price of the underlying asset



Valuing the Portfolio

(Risk-Free Rate is $1/20 = 5\%$)

- The riskless portfolio is:
 - long $1/2$ shares
 - short 1 call option
- The value of the portfolio next period is
$$32 \times (1/2) - 7 = 18 \times (1/2) = 9$$
- The value of the portfolio today is
$$9 / (1 + 1/8) = 9 / (9/8) = 8$$

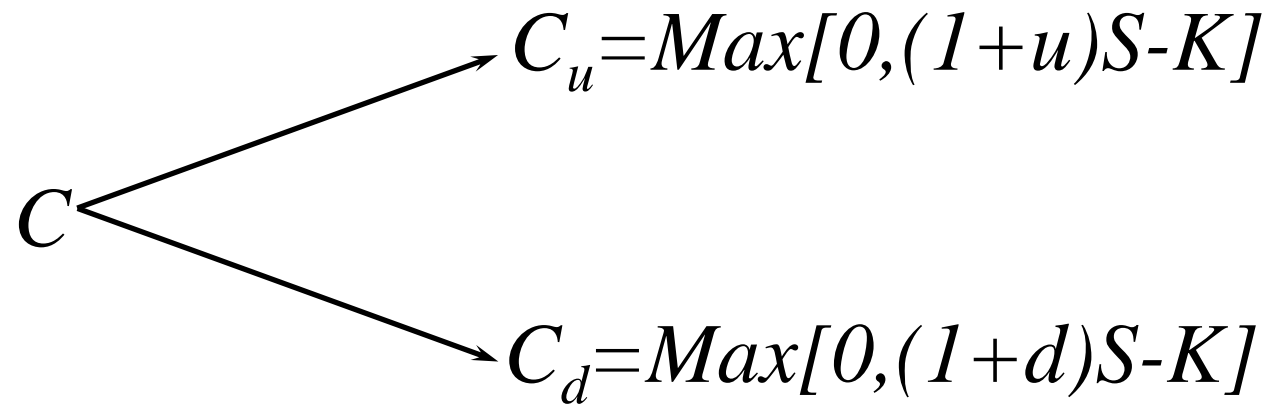
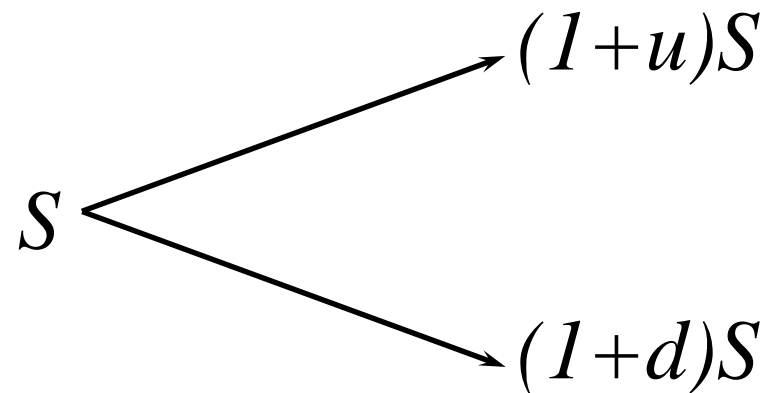


Valuing the Option

- The portfolio that is
long $1/2$ shares
short 1 option
is worth 8 today
- The value of the shares is
 $12 (= 24/2)$
- The price of the option therefore satisfies
 $12 - c = 8$ or $c = 4$

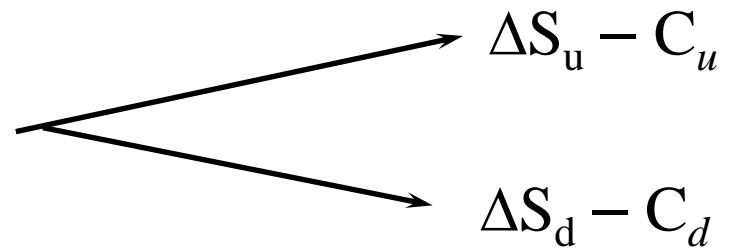


Generalisation



Generaliation (continued)

- Consider the portfolio that is long Δ shares and short 1 call



- The portfolio is riskless when $\Delta S_u - C_u = \Delta S_d - C_d$ or

$$\Delta = \frac{C_u - C_d}{S_u - S_d}$$



Generalization

(continued)

- Value of the portfolio at time T is

$$\Delta S_u - C_u$$

- Value of the portfolio today is

$$(\Delta S_u - C_u)/(1+r)$$

- Another expression for the portfolio value today is $\Delta S - C$

- Hence

$$C = \Delta S - (\Delta S_u - C_u)/(1+r)$$



Interpreting Δ :

- Delta, Δ , is the riskless **hedge** ratio; $0 \leq \Delta_c \leq 1$.
- Delta, Δ , is the number of shares needed to hedge one call. I.e., if you are long one call, you can hedge your risk by selling Δ shares of stock.
- Therefore, the number of calls to hedge one share is $1/\Delta$. I.e., if you own 100 shares of stock, then sell $100/\Delta$ calls to hedge your position. Equivalently, buy Δ shares of stock and write one call.
- Delta is the difference in the call price ($C_u - C_d$) divided by the difference stock price ($S_u - S_d$)
- In continuous case, $\Delta = \partial C / \partial S$ = the change in the value of a call caused by a (small) change in the price of the underlying asset.



Generalization

(continued)

- Substituting for Δ and $S_u=(1+u)S$, $S_d=(1+d)S$ we obtain the following call price formula

$$C = [p C_u + (1 - p) C_d] / (1+r)$$

where

$$p = \frac{r - d}{u - d}$$



Using p

$$p = \frac{r - d}{u - d}$$

- In the example with $r = 1/8$, $d = -1/4$, $u = 1/3$, then $p = 9/14$.

- In our example, if p is the probability of an uptick then

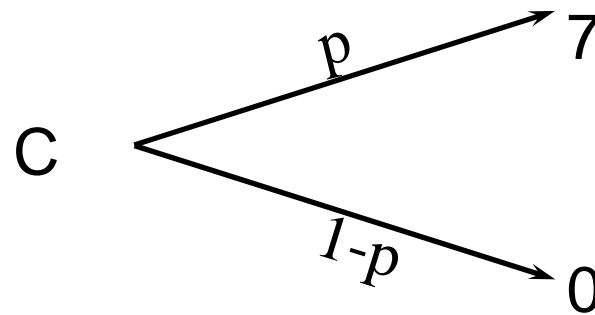
$$\begin{aligned} S_{T-1} &= [(p)(32) + (1-p)(18)]/(9/8) \\ &= [(9/14)(32) + (5/14)(18)]/(9/8) \\ &= 8[(32/14) + (10/14)] = 24. \end{aligned}$$

- Find p from the equation

$$24(9/8) = 32p + 18(1-p) = 18 + 14p$$



Using p to value the option



- $C = [(9/14)(7) + (5/14)(0)] / (9/8) = [9/2] / (9/8) = 8/2 = 4!$



Interpreting p

$$p = \frac{r - d}{u - d}$$

- p is the probability of an uptick in a **risk-neutral world**.
- In a **risk-neutral world**, all assets (including the stock and the option) will be priced to provide the same expected rate of return, r.
- That is, the stock is priced to provide the same expected rate of return as the call option

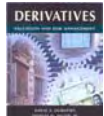


Option Elasticity (Omega)

$$\Delta = \frac{C_u - C_d}{(u - d)S} = \left(\frac{dC}{dS} \right)$$

$$\Omega_C = \left(\frac{S}{C} \right) \Delta \geq 1$$

$$\Omega_P = \left(\frac{S}{P} \right) \Delta < 0$$



Option Risk

- The expected rate of return on a call is

$$\mu_C = (\pi C_u + (1-\pi)C_d - C)/C$$

- Return on the call in the up state is $(7-4)/4 = 3/4$ (75%).
- Return on call in the down state is $(0-4)/4 = -1$ (-100%).
- Suppose $\pi = 3/4$. Expected return on call is

$$\mu_C = ((3/4)(75) + (1/4)(-100)) = 31.25\%$$

- Expected return on the stock is

$$\mu_S = ((3/4)(33.33) + (1/4)(-25)) = 18.75\%$$



Option Risk (2)

- The standard deviation of the stock is

$$\sigma_S = \sqrt{\left(\frac{3}{4}\right)(33.33-18.75)^2 + \left(\frac{1}{4}\right)(-25-18.75)^2} = 26.29\%$$

- The standard deviation of the call option is

$$\sigma_C = \sqrt{\left(\frac{3}{4}\right)(75-31.25)^2 + \left(\frac{1}{4}\right)(-100-31.25)^2} = 78.87\%$$

- The ratio is $\sigma_C/\sigma_S = 3$.



Excess Returns

- For a call option

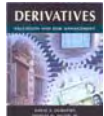
$$\mu_C - r = \Omega_C(\mu_S - r)$$

- In our example

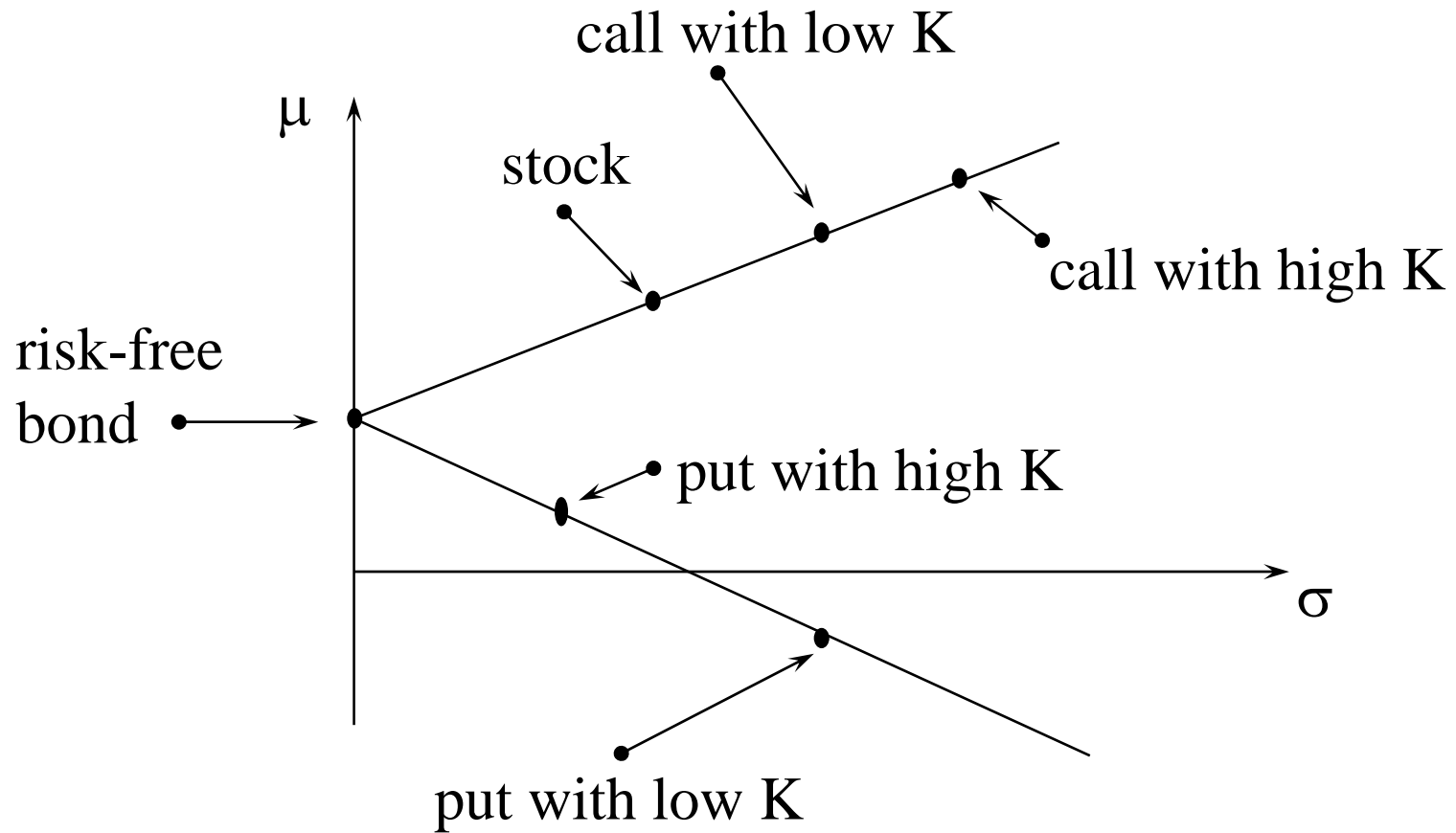
$$\mu_C - r = 31.25 - 12.5 = 18.75$$

$$\mu_S - r = 18.75 - 12.5 = 6.25$$

$$\Omega_C = 3$$



Risk-Return



Option Beta

- Capital Asset Pricing Model

$$\mu_S - r = \beta_S(\mu_M - r)$$

- From above

$$\mu_C - r = \Omega_C(\mu_S - r)$$

- Call option beta

$$\beta_C = \Omega_C\beta_S > \beta_S$$

- CAPM for call option

$$\mu_C - r = \Omega_C\beta_S(\mu_M - r)$$



Option Beta (Example)

- Suppose the expected return on the market is 15%

$$\mu_S - r = \beta_S(\mu_M - r)$$

$$6.25 = \beta_S(15 - 12.5) = \beta_S(2.5)$$

$$\beta_S = 2.5$$

- Call option beta

$$\beta_C = \Omega_C \beta_S = 3(2.5) = 7.5$$

- Check

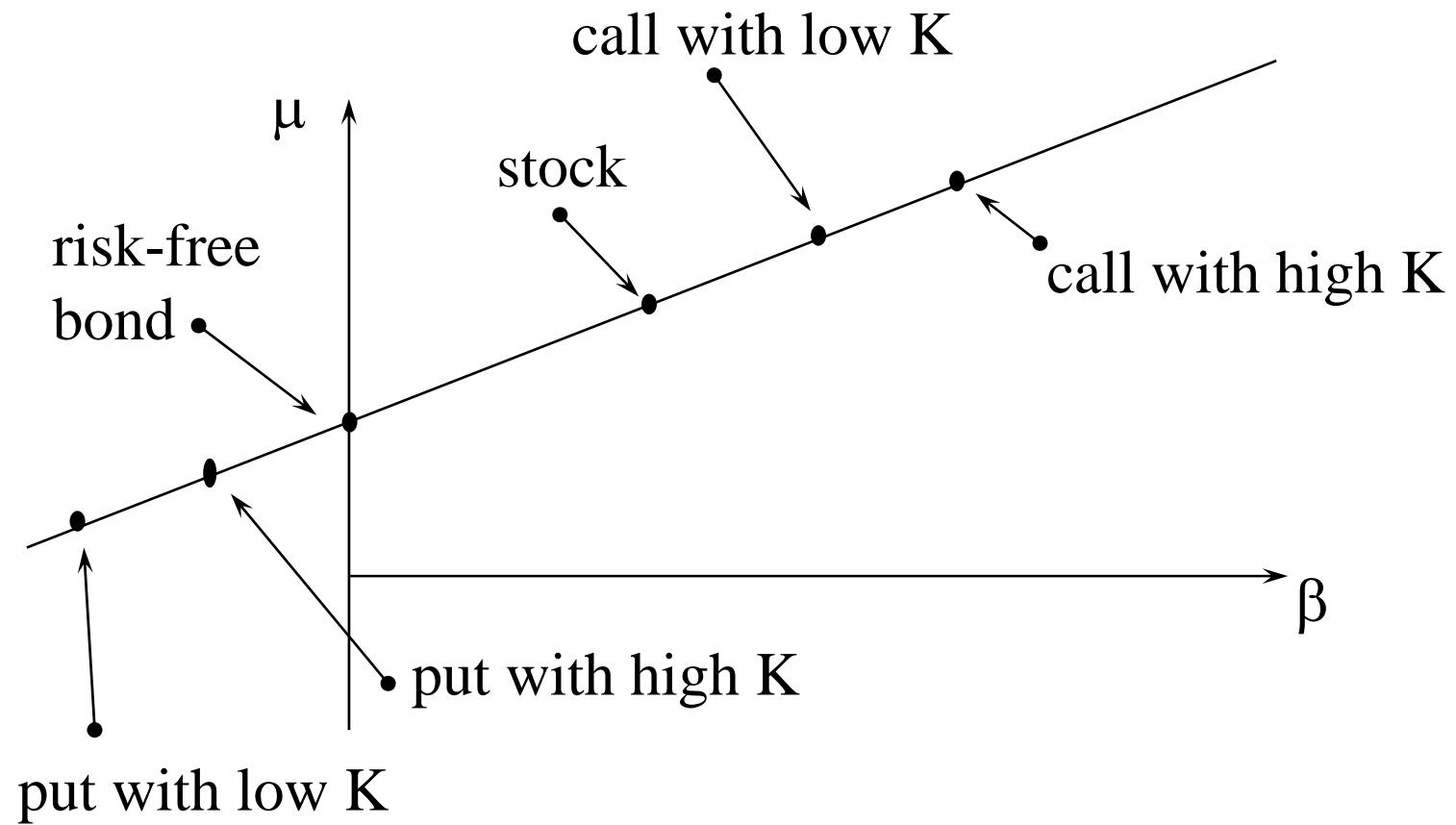
$$\mu_C - r = \beta_C(\mu_M - r)$$

$$18.75 = \beta_C(2.5)$$

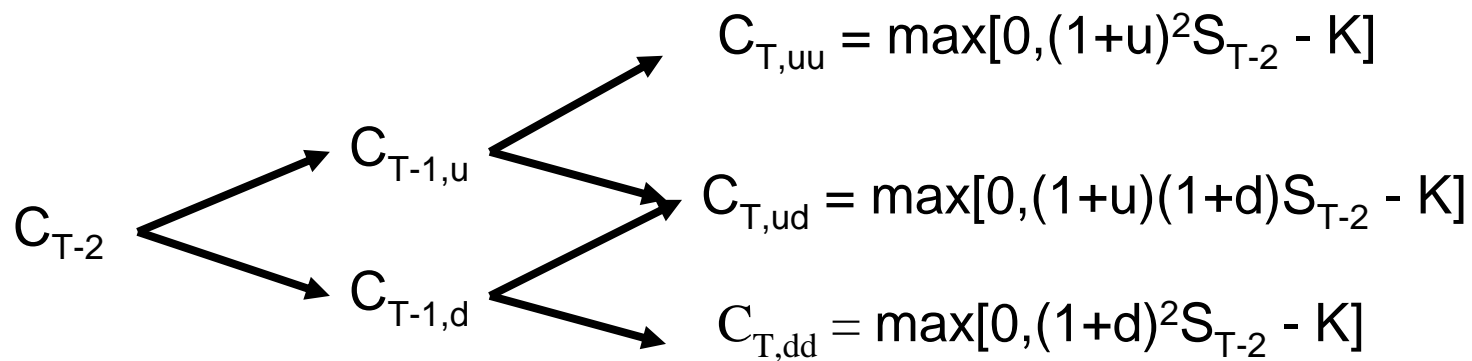
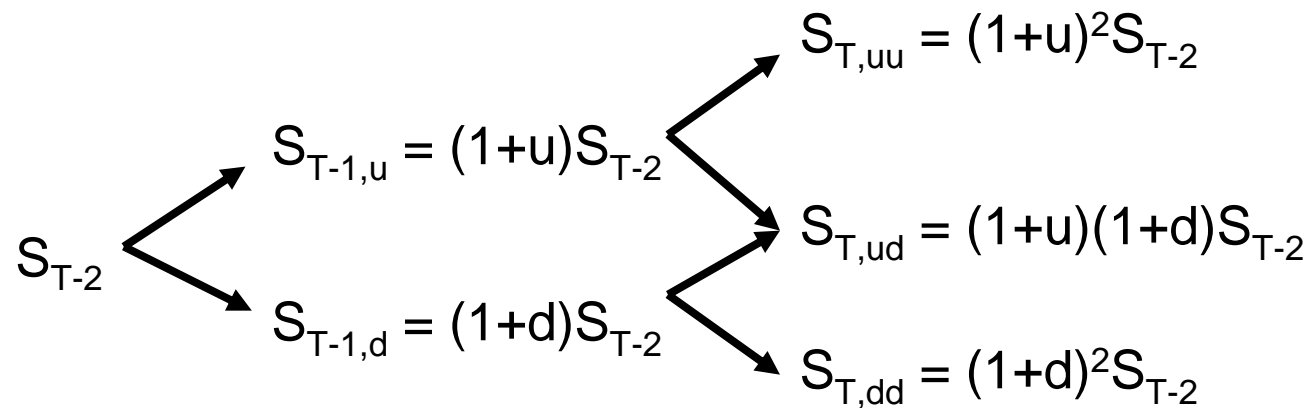
$$\beta_C = 7.5$$



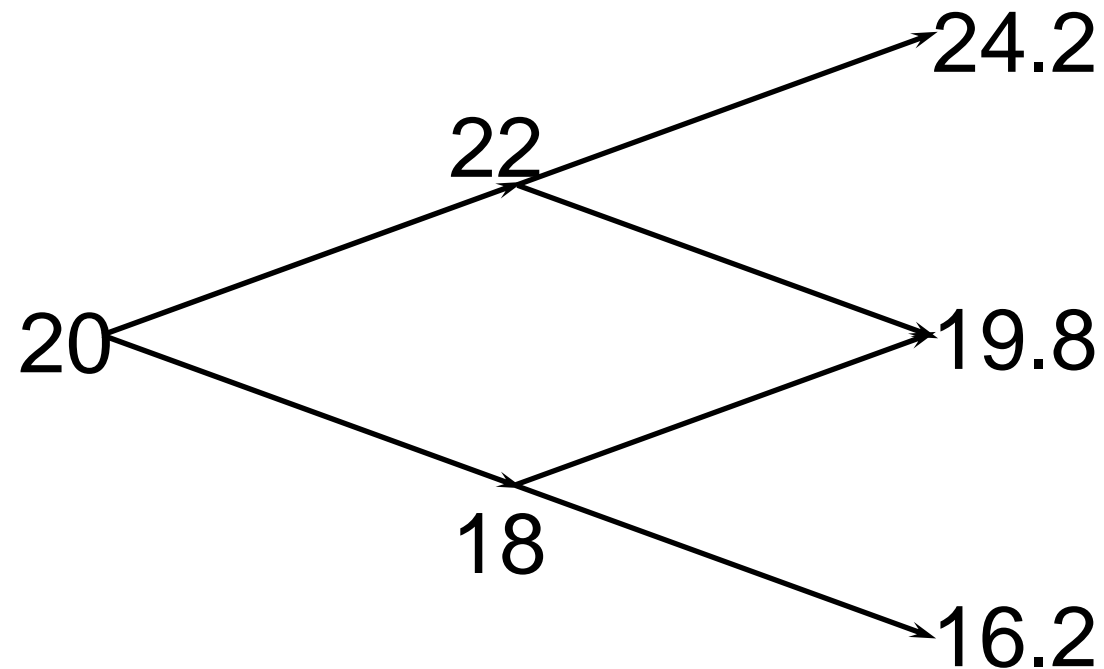
Security Market Line



Two Period Binomial Model



A Two-Step Example



- Each time step is 1 period
- $r = 0.05$, $u = 0.1$, $d = -0.1$, so $p = \frac{3}{4}$.



Call Formula for one Period

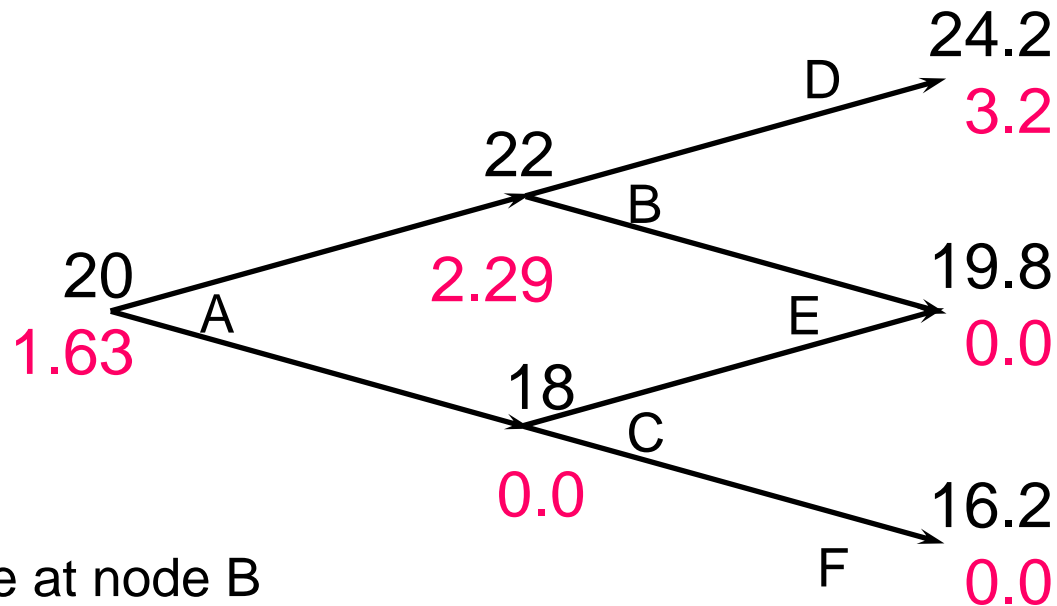
$$c = [p c_u + (1 - p) c_d] / (1 + r)$$

where

$$p = \frac{r - d}{u - d}$$



Valuing a Call Option (K=21)



- Value at node B
 $= ((3/4) \times (16/5) + (1/4) \times 0) / (21/20) = 16/7 = 2.29$
- Value at node A
 $= ((3/4) \times (16/7) + (1/4) \times 0) / (21/20) = 80/49 = 1.63$



Call Formula for two periods

$$c = [p^2 c_{uu} + 2p(1-p)c_{ud} + (1 - p)^2 c_{dd}] / (1+r)^2$$

where

$$c_{uu} = \text{Max}[0, (1+u)^2 S - K];$$

$$c_{ud} = \text{Max}[0, (1+u)(1+d)S - K];$$

$$c_{dd} = \text{Max}[0, (1+d)^2 S - K];$$

$$p = \frac{r - d}{u - d}$$



Dynamic Delta Hedging

- In the one period model

$$\Delta = (C_u - C_d)/(u-d)S = (1-0)/(22-18) = 1/4$$

- In the two period model (1st period)

$$\Delta = (16/7)/(22-18) = 4/7$$

- In the two period model (2nd period if stock goes up in 1st period)

$$\Delta = (3.2-0)/(24.2-19.8) = 8/11$$

- In the two period model (2nd period if stock goes down in 1st period)

$$\Delta = 0-0/(19.8-16) = 0$$



The Strategy is Self Financing

- At node A, $\Delta = 4/7$ and $B = -480/49$.
- At node B selling the shares and repaying the loan gives
 $(4/7)(22) - (480/49)(21/20) = (88/7) - (72/7) = 16/7$.
- At node C the value of the portfolio is
 $18(4/7) - (480/49)(21/20) = (72/7) - (72/7) = 0$.
- At node B we choose $\Delta = 8/11$ and $B = -96/7$ and the net cost of this portfolio is
 $(8/11)22 - (96/7) = (112/7) - (96/7) = 16/7$.
- Thus buying the new portfolio at node B requires no addition in flow of cash. The strategy is *self-financing*.



The Multi-Period BOPM

- We can find binomial option prices for **any number of periods** by using the following five steps:
 - (1) Build a price “tree” for the underlying.
 - (2) Calculate the possible option values in the last period (time T = expiration date)
 - (3) Calculate all possible option prices in the penultimate period using one of the three methods we know.
 - (4) Keep working back through the tree to “Today” (Time $T-n$ in an n -period, $(n+1)$ -date, model).



The 'n' Period Binomial Formula:

If $n = 3$:

$$C_{T-3} = \frac{p^3 C_{T,uuu} + 3p^2(1-p)C_{T,uud} + 3p(1-p)^2 C_{T,udd} + (1-p)^3 C_{T,ddd}}{(1+r)^3} \quad (17-15)$$

The “binomial coefficient” computes the number of ways we can get j upticks in n periods:

$$\binom{n}{j} = \frac{n!}{j!(n-j)!}$$

Thus, the 3-period model can be written as:

$$C_{T-3} = \frac{1}{(1+r)^3} \sum_{j=0}^3 \binom{3}{j} p^j (1-p)^{3-j} \max[0, (1+u)^j (1+d)^{3-j} S_{T-3} - K].$$



The 'n' Period Binomial Formula:

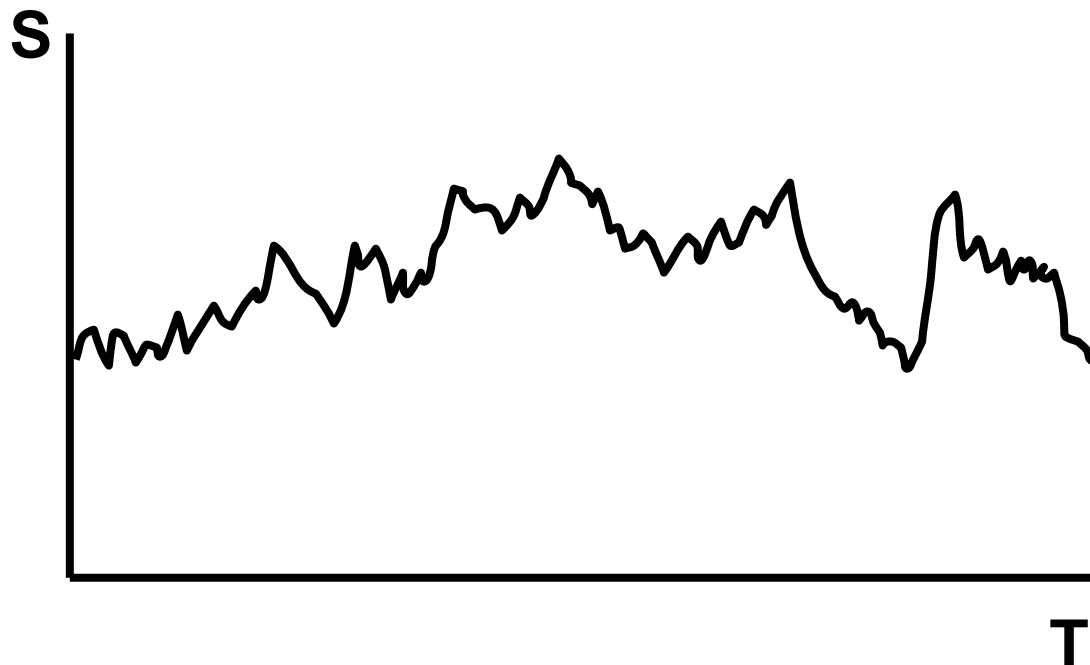
In general, the n-period model is:

$$C = \frac{1}{(1+r)^n} \sum_{j=a}^n \binom{n}{j} p^j (1-p)^{n-j} [(1+u)^j (1+d)^{n-j} S_{T-n} - K] \quad (17-17)$$

Where “a” in the summation is the minimum number of up-ticks so that the call finishes in-the-money.



Suppose the Number of Periods Approaches Infinity



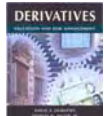
In the limit, that is, as N gets 'large', and if u and d are chosen in just the right way then the BOPM converges to the Black-Scholes Option Pricing Model (the BSOPM is the subject of Chapter 18).



Chapter 18

Continuous Time Option Pricing Models

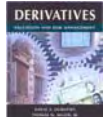
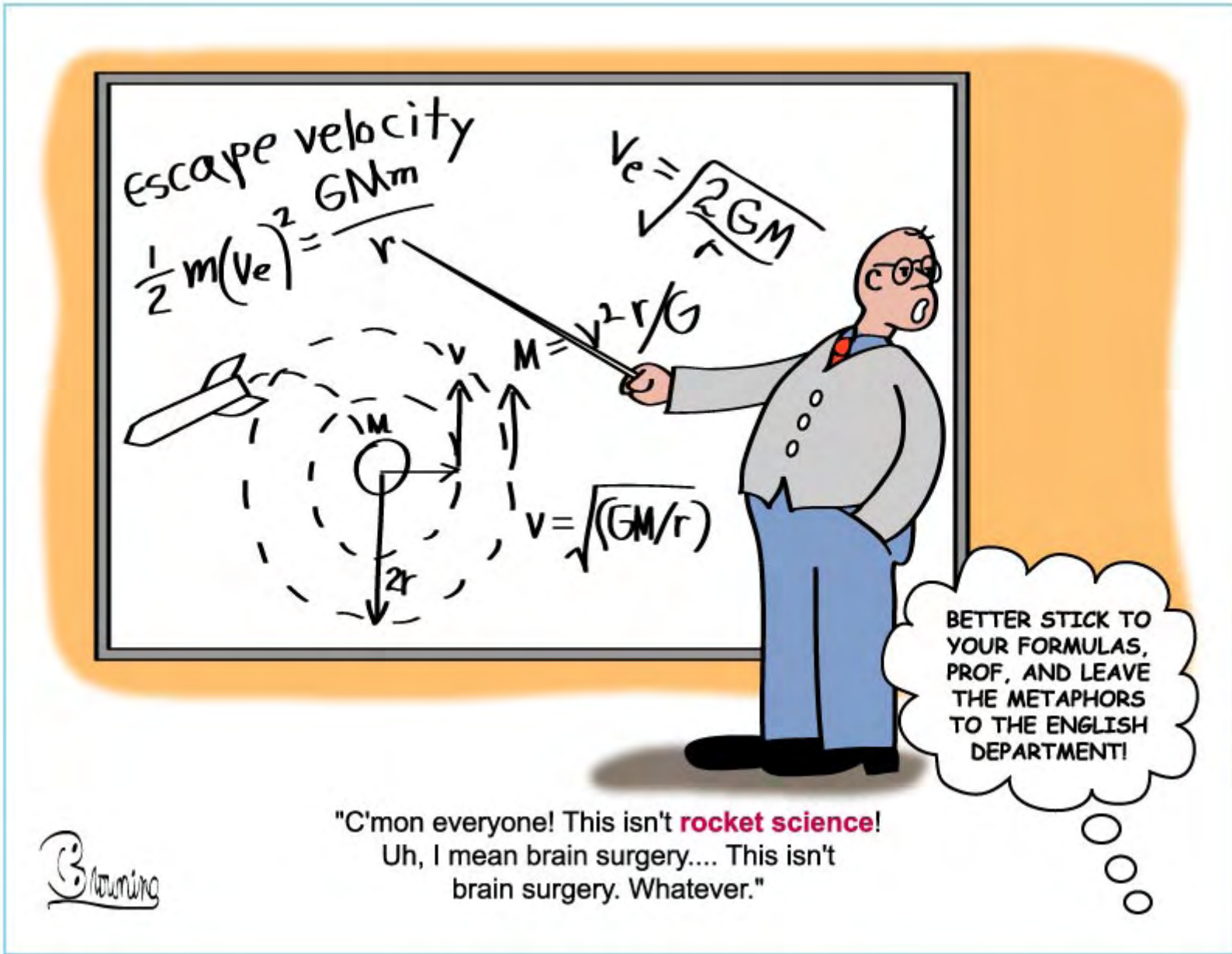
- Assumptions of the Black-Scholes Option Pricing Model (BSOPM):
 - Perfect Markets - No taxes, No transactions costs, Unrestricted short-selling of stock, with full use of short-sale proceeds, Shares are infinitely divisible
 - Constant riskless interest rate for borrowing/lending
 - Continuous trading
 - The stock price evolves via a specific 'process' through time (more on this later....)



Derivation of the BSOPM

- Specify a ‘process’ that the stock price will follow (i.e., all possible “paths”)
- Construct a riskless portfolio of:
 - Long Call
 - Short Δ Shares
 - (or, long Δ Shares and write one call, or long 1 share and write $1/\Delta$ calls)
- As delta changes (because time passes and/or S changes), one must maintain this risk-free portfolio over time
- This is accomplished by purchasing or selling the appropriate number of shares.





The BSOPM Formula

$$C = SN(d_1) - Ke^{-rT}N(d_2)$$

where $N(d_i)$ = the cumulative standard normal distribution function, evaluated at d_i , and:

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$N(-d_i) = 1 - N(d_i)$$



What we need to know

- Continuous compounding
 - Continuously compounded rates of return
- Stochastic process for stock prices
 - Geometric Brownian motion
- Properties of Normal distribution
 - Lognormal distribution



Compound Interest

- £A invested at risk-free rate after T periods

$$A(1+r)^T$$

- If interest is compounded semi-annually

$$A(1+(r/2))^{2T}$$

- If interest is compounded quarterly

$$A(1+(r/4))^{4T}$$

- Example: $A = 100$; $r = 0.1$

£110 (annual)
(quarterly)

£110.25 (semi-annual)

£110.38



The Rate of Return

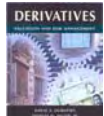
- If an initial investment costs B_0 and the final payoff is B_T , the rate of return is

$$\rho = (B_T - B_0) / B_0$$

- Or equivalently ρ satisfies

$$B_T = B_0(1 + \rho)$$

- Example: $B_T = 120$, $B_0 = 100$, then $\rho = 1/5$ (20%)
- To compare rates of return over different time periods we need to measure the rate of return per period, say an annualised rate of return r .



Compound Interest Rates

- Annualising the interest rate can be done by compounding. If this is done n times, then r satisfies

$$(1 + \rho) = (1 + r/n)^{nT}$$

- Example $r = 20\%$, $T=2$.

With annual compounding ($n = 1$)

$$(1 + \rho) = (1 + r)^2 \quad \text{so } r = 0.09544$$

With semi-annual compounding ($n=2$)

$$(1 + \rho) = (1 + r/2)^4 \quad \text{so } r = 0.09327$$

With quarterly compounding ($n=4$)

$$(1 + \rho) = (1 + r/4)^8 \quad \text{so } r = 0.0922075$$



Continuous compounding

- Let n increase to infinity

$$(1 + \rho) = (1 + r/n)^{nT} \rightarrow e^{rT}$$

where

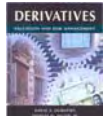
$$e = 2.7182818$$

is the base of the natural logarithm

- Since $(1 + \rho) = B_T/B_0$

$$r = (1/T)(\ln B_T - \ln B_0)$$

- This value of r is known as the *continuously compounded rate of return*.
- Example $T = 2$, $B_T = 120$, $B_0 = 100$. Then $r = 0.91161$.



Continuously compounded returns are nice!

- Let r_t denote the continuously compounded rate of return from $t-1$ to t , that is

$$r_t = \ln B_t - \ln B_{t-1}$$

- Let $r(T)$ be the continuously compounded return over T periods, that is

$$r(T) = \ln B_T - \ln B_0$$

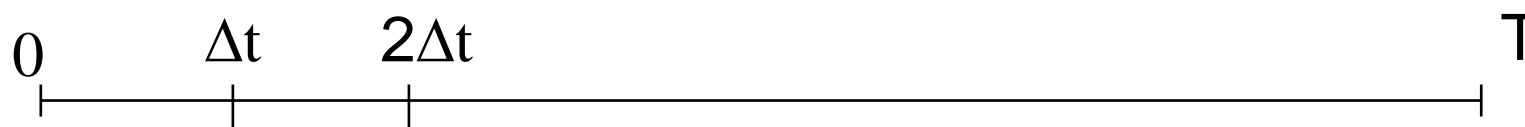
- Then

$$r(T) = r_1 + r_2 + r_3 + \dots + r_{T-1} + r_T$$

We find the continuously compounded rate of return over time T is the *sum* of the period by period returns.



Differential Equations



$$S_{t+\Delta t} = (1 + r\Delta t)S_t$$

$$\frac{S_{t+\Delta t} - S_t}{S_t \Delta t} = r$$

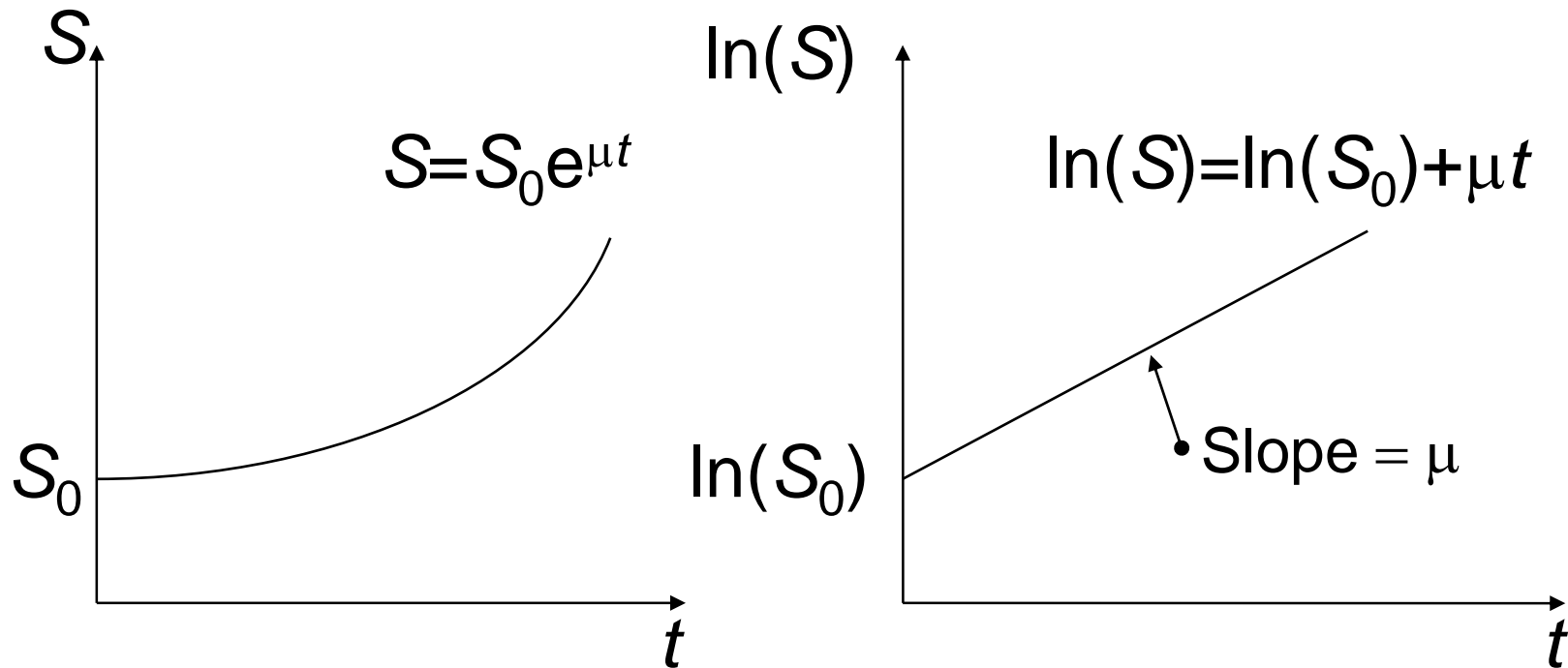
$$\lim_{\Delta t \rightarrow 0} \frac{S_{t+\Delta t} - S_t}{S_t \Delta t} = \frac{dS}{S dt} = r$$

$$dS = rS dt$$



Deterministic Component

$$dS = \mu S dt$$



Forward Contract

- Value of a forward contract with delivery price of X :

$$f(S_t, t) = S_t - Xe^{-r(T-t)}$$

- Variables: S_t, t
- Parameters: X, r, T .
- Boundary Condition:

$$f(S_T, T) = S_T - X$$



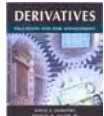
Differential Equation for Forward Contract

- Partial Derivatives

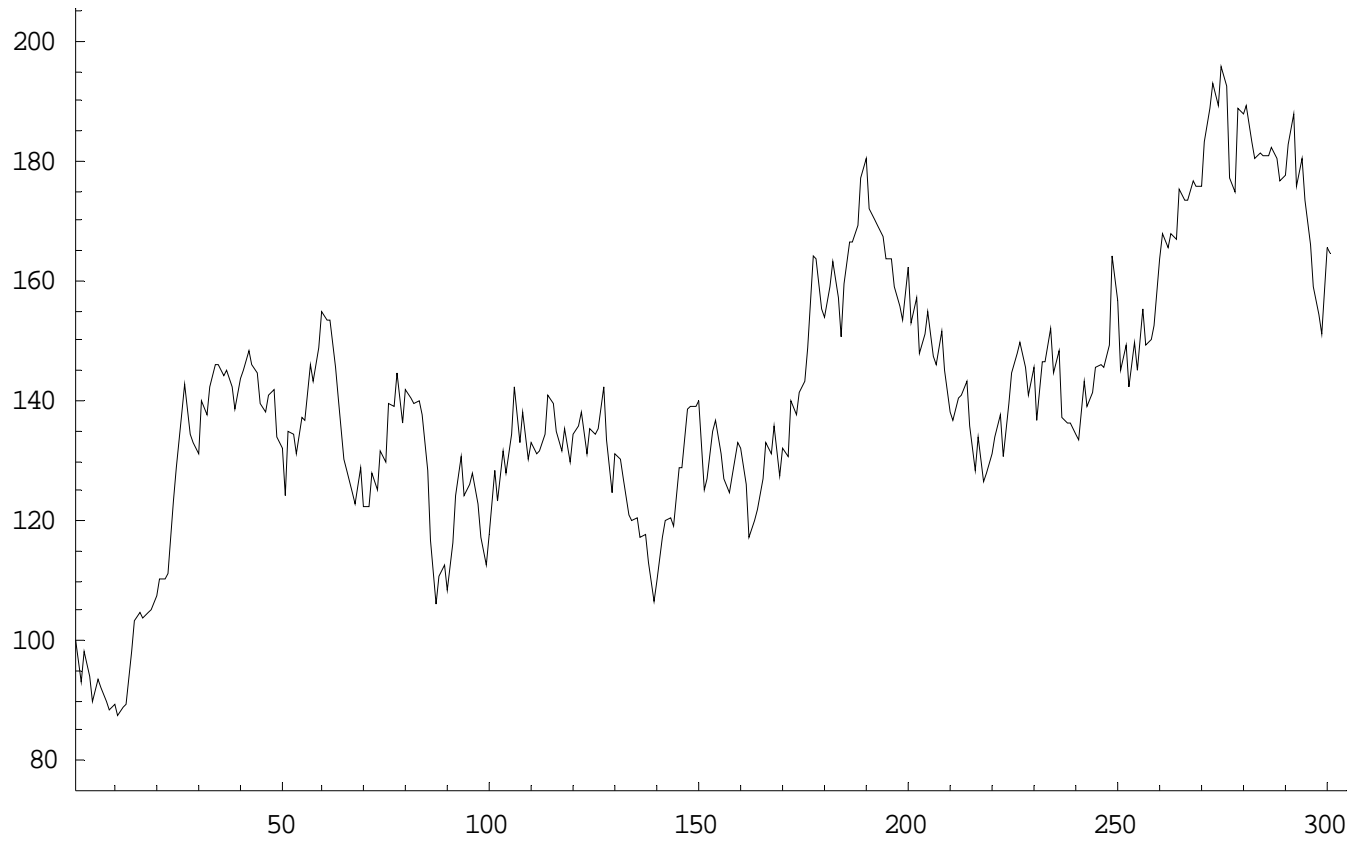
$$\frac{\partial f(S_t, t)}{\partial S_t} = 1; \quad \frac{\partial f(S_t, t)}{\partial t} = -rXe^{-r(T-t)}.$$

- Differential Equation

$$df(S_t, t) = dS_t - r(S_t - f(S_t, t))dt$$



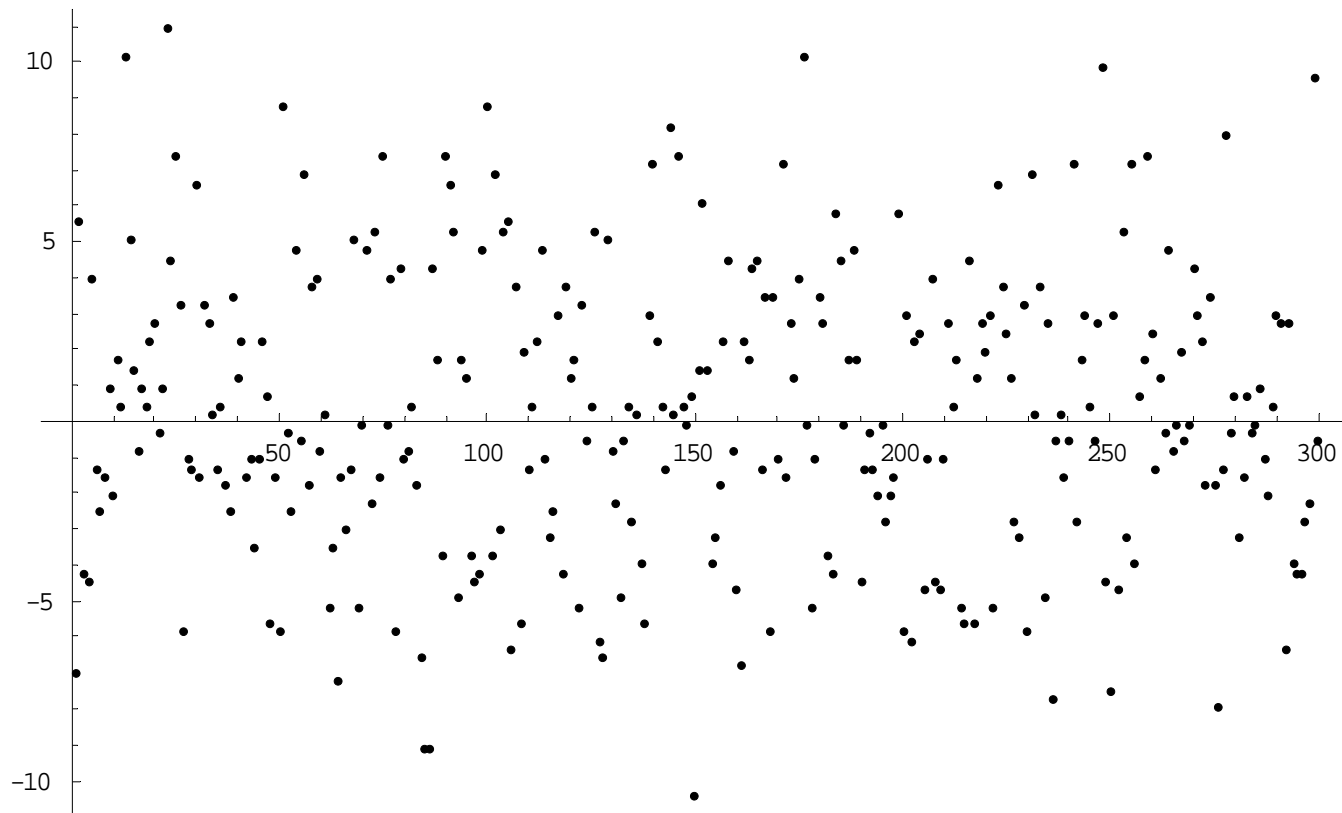
Weekly Price of ABC Stock



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Thomas W. Miller, Jr.



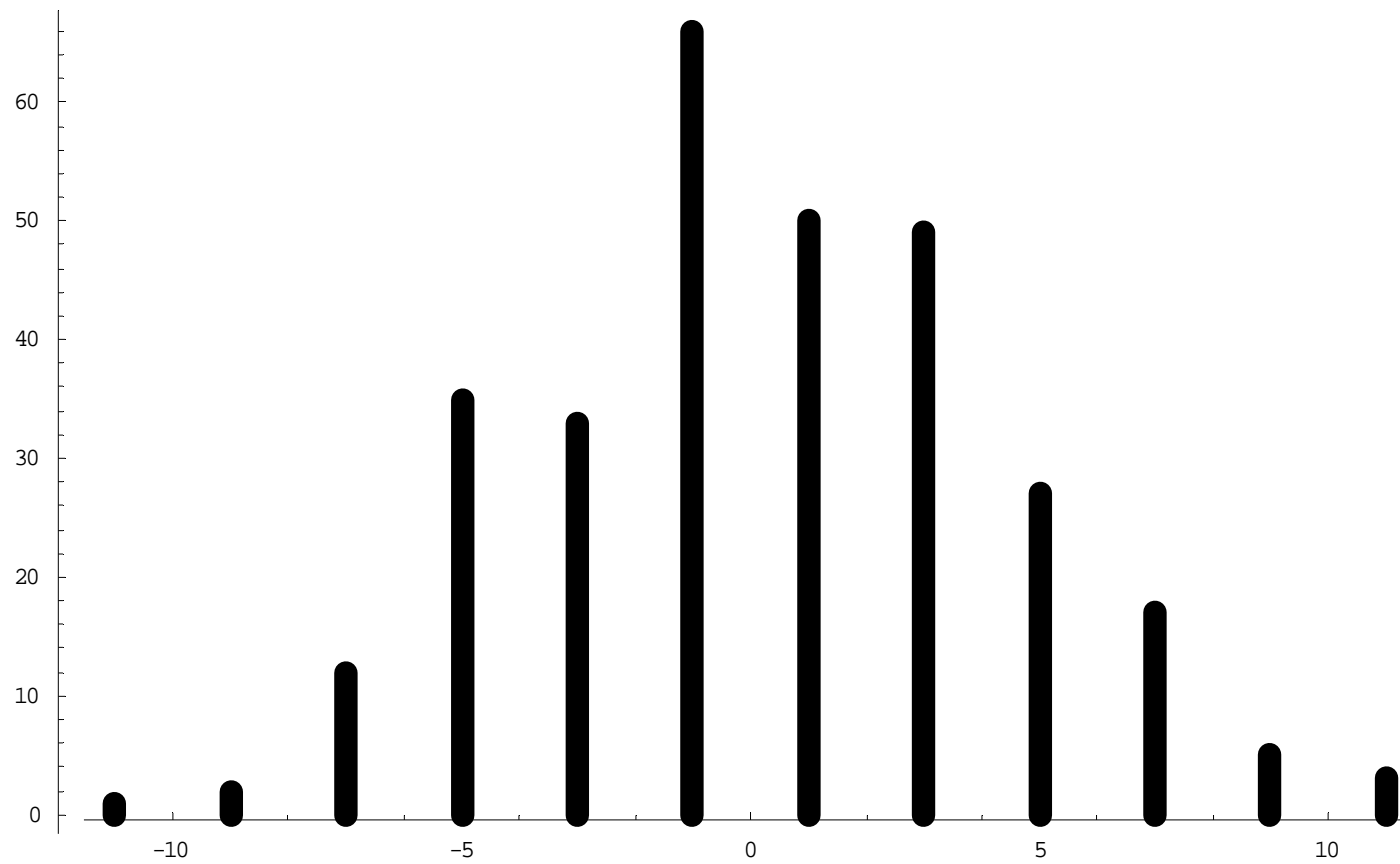
Weekly Returns on ABC Stock



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Histogram for ABC Stock



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Bachelier's Square Root Rule

- The size of stock price fluctuations grows larger with the time horizon
- In a minute fluctuations mostly less than one tick point
- over a day much larger
- over a year larger still
- the range of fluctuations will be “proportional to the square root of time”

(Bachelier)



The Efficient Markets Hypothesis

- No abnormal profits
 - if the price were sure to go up it already would have done so
 - all information rapidly assimilated into prices
- Only new information (news) causes prices to change
 - news is unforecastable
 - so too are prices

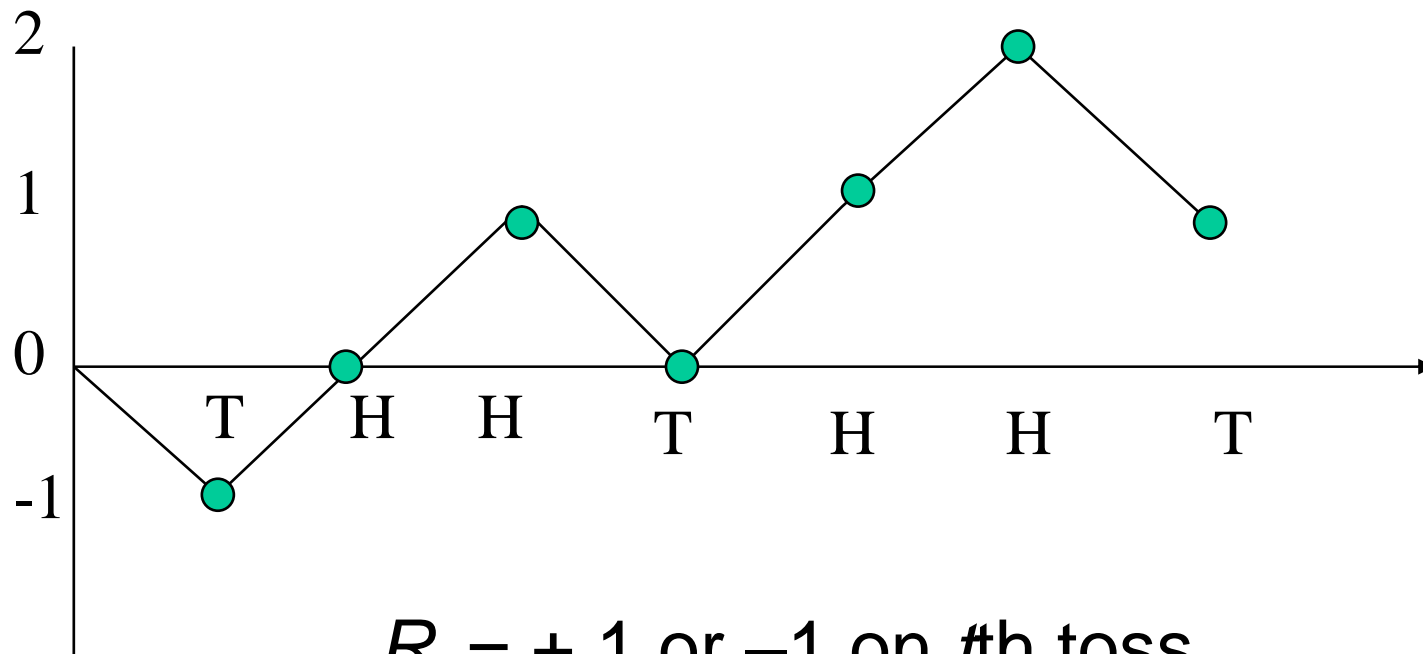


Consequences of EMH

- stock prices follow a Markov process
 - In a Markov process future movements in a variable depend only on where we are, not the history of how we got where we are
 - With (weak-form) market efficiency it is impossible to produce consistently superior returns with a trading rule based on the past history of stock prices.



Coin Tossing

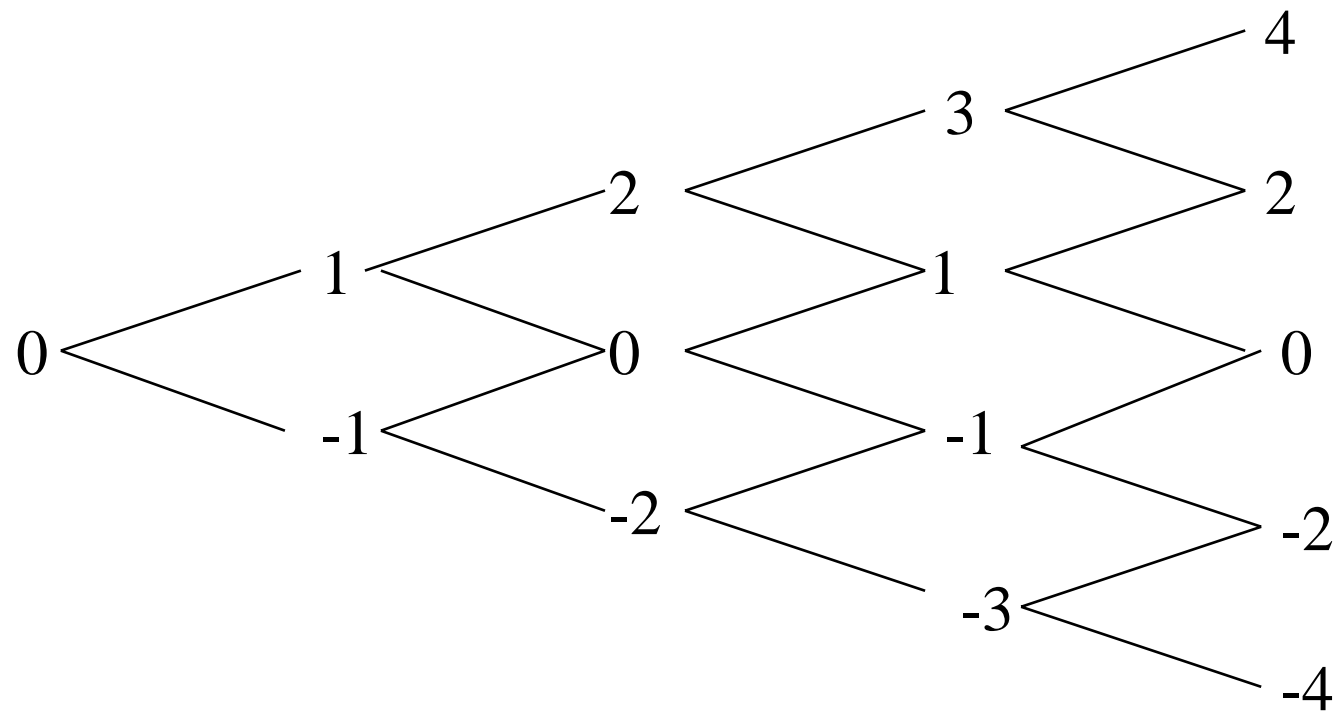


$R_t = +1$ or -1 on t th toss

$$E[R_t] = 0; E[R_t^2] = 1; E[R_t R_k] = 0$$



Coin Tossing (2) ($\pi = \frac{1}{2}$)

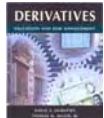


$$\sigma^2 = 1$$

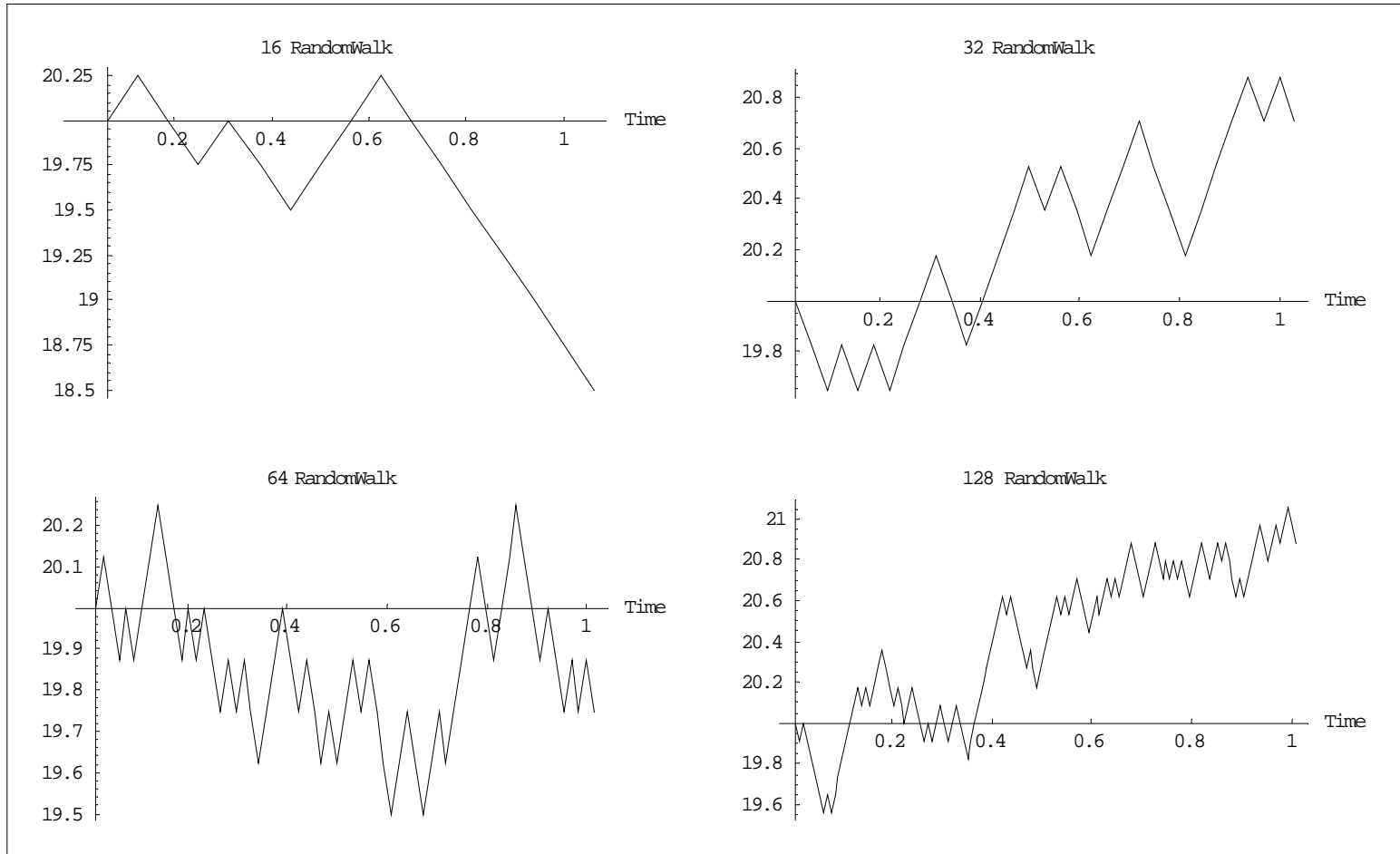
$$\sigma^2 = 2$$

$$\sigma^2 = 3$$

$$\sigma^2 = 4$$

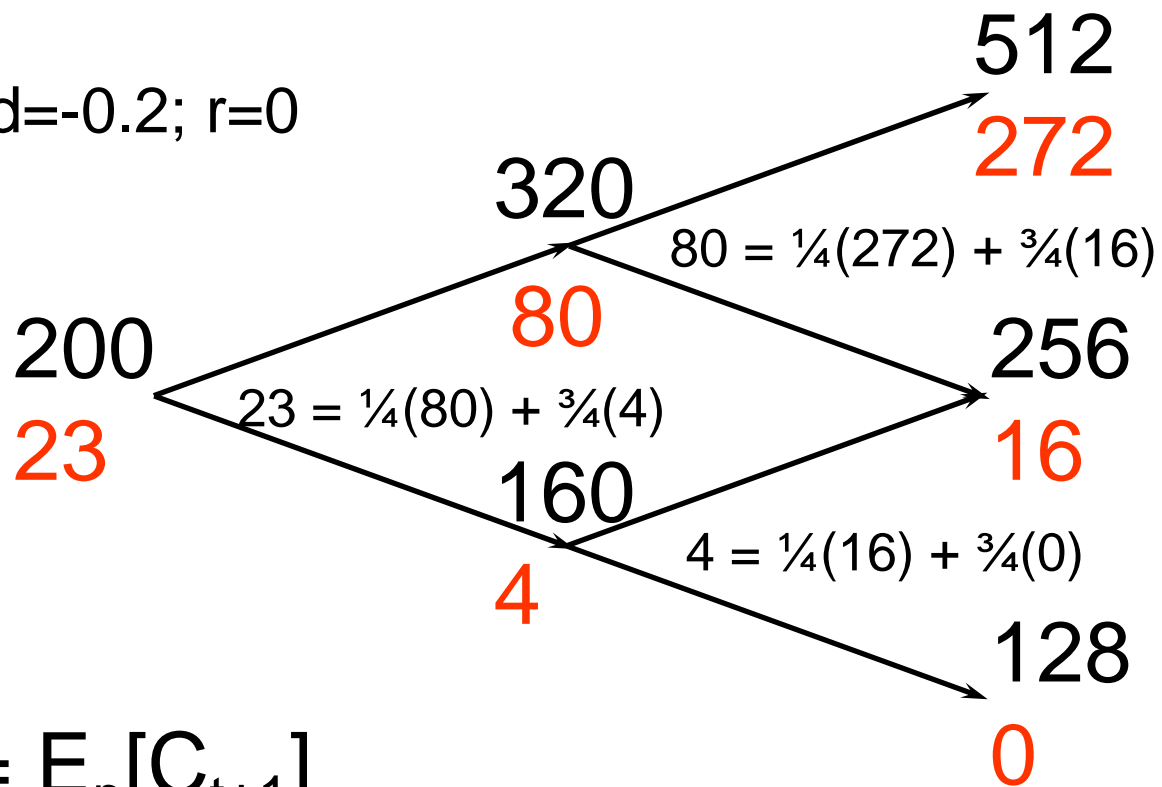


Random Walk



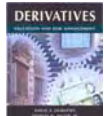
Call (K = 240)

$u=0.6; d=-0.2; r=0$

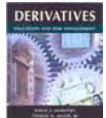
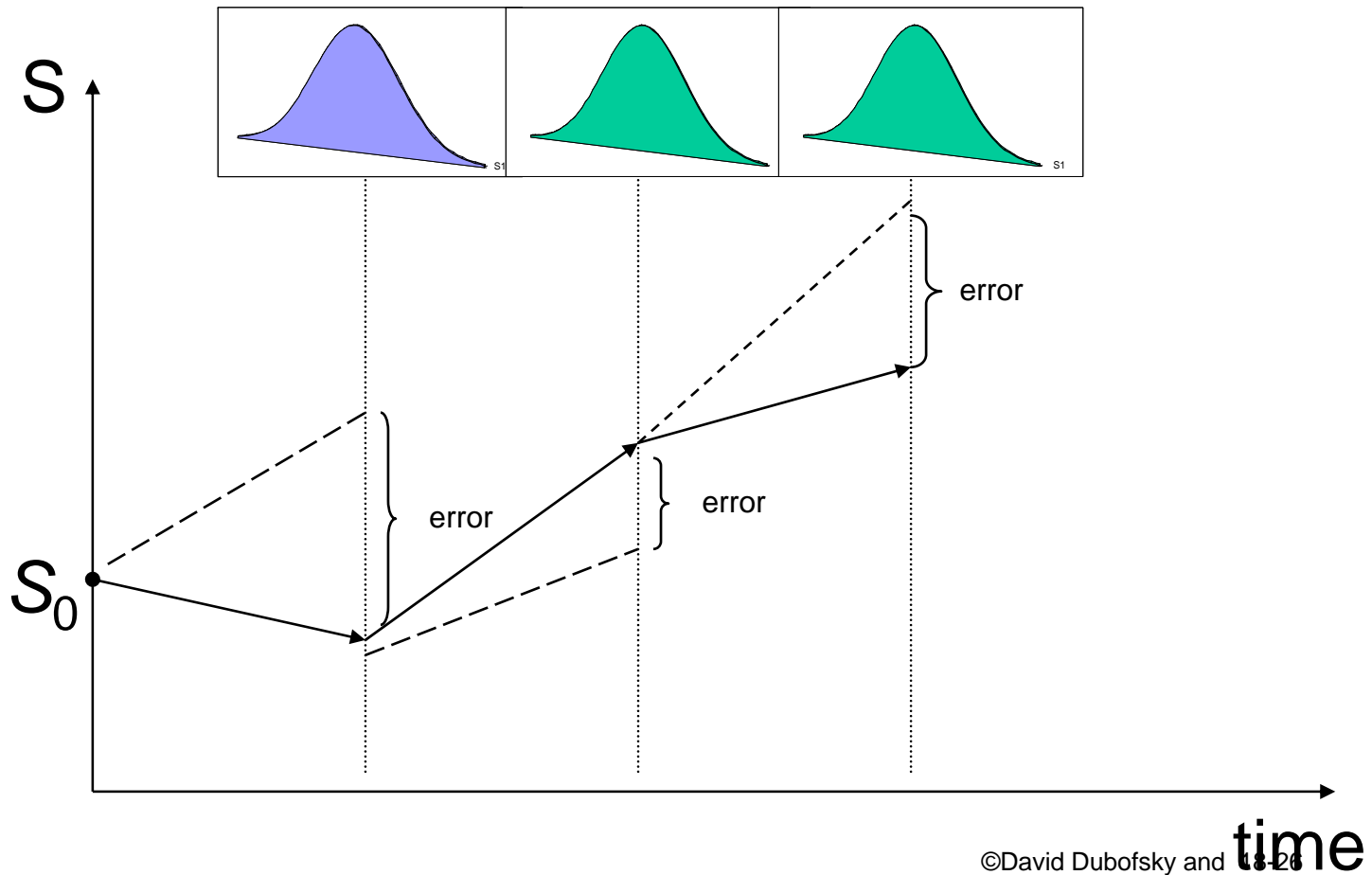


Summary

- The EMH implies that stock prices follow a Markov process
- Empirically the standard deviation in stock prices is proportional to the square root of time
- Random Walk is a good approximation
- Risk-neutral probabilities form a martingale measure for stocks and derivatives

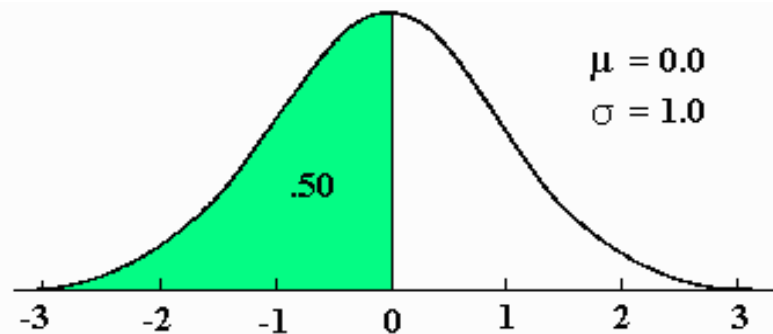


Random Component



The Standard Normal Curve

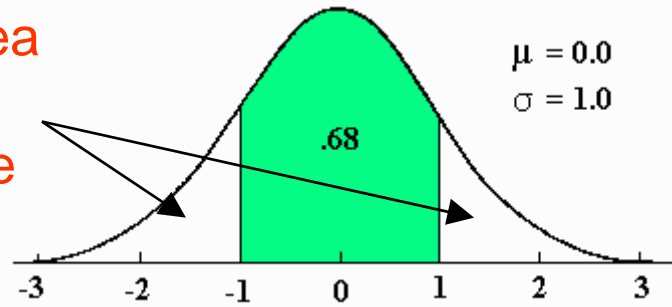
- The standard normal curve is a member of the family of normal curves with $\mu = 0.0$ and $\sigma = 1.0$. The X-axis on a standard normal curve is often relabeled and called 'Z' scores.
- The area under the curve equals 1.0.
- The cumulative probability measures the area to the left of a value of Z. E.g., $N(0) = \text{Prob}(Z) \leq 0.0 = 0.50$, because the normal distribution is symmetric.



The Standard Normal Curve

The area between Z-scores of -1.00 and +1.00 is 0.68 or 68%.

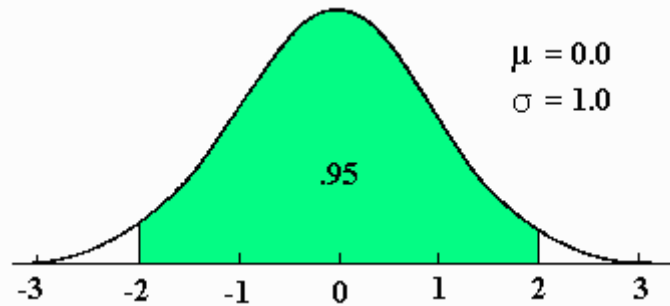
Note also that the area to the left of $Z = -1$ equals the area to the right of $Z = 1$. This holds for any Z .



What is $N(1)$?
What is $N(-1)$?

The area between Z-scores of -1.96 and +1.96 is 0.95 or 95%.

What is the area in each tail? What is $N(1.96)$? What is $N(-1.96)$?

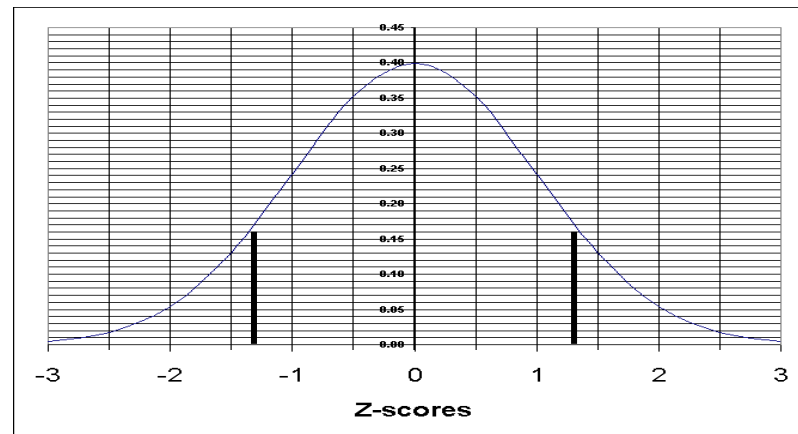


Cumulative Normal Table (Pages 560-561)

Cumulative Probability for the Standard Normal Distribution

z	2nd digit of z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

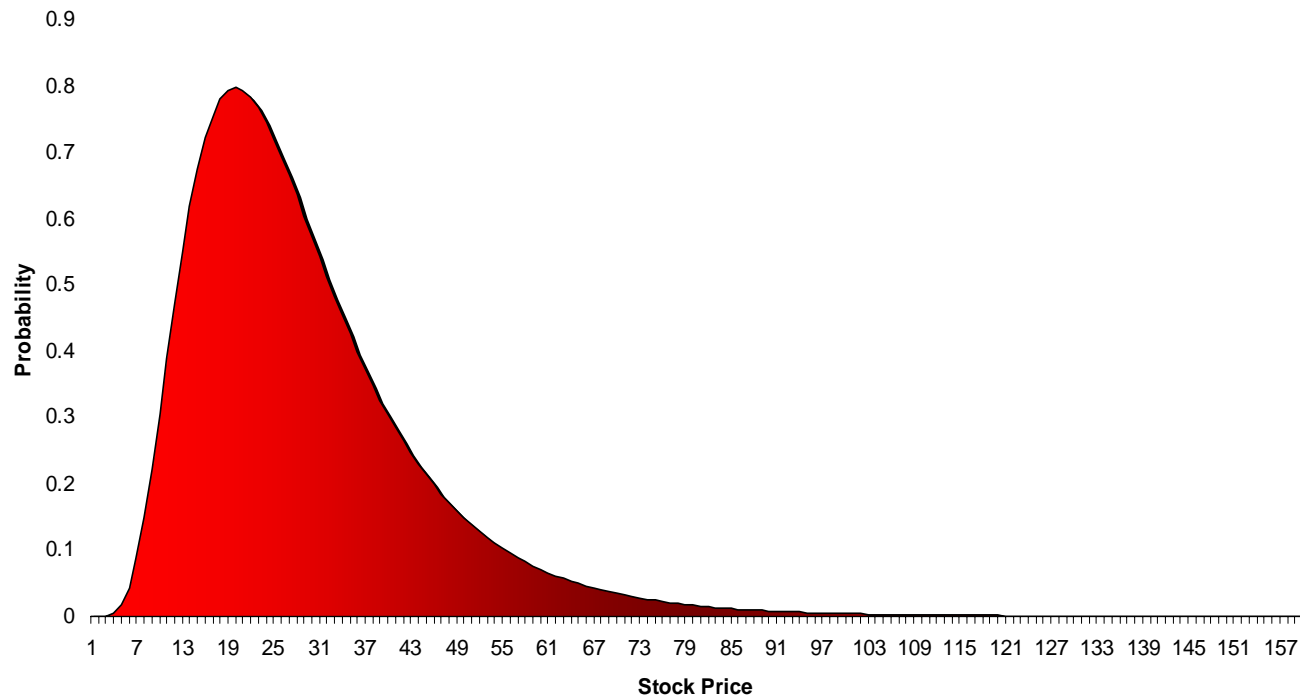
$$N(-d) = 1 - N(d)$$



$$N(-1.34) = 1 - N(1.34)$$

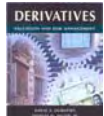


The Lognormal Distribution



$$E(S_T) = S_0 e^{\mu T}$$

$$\text{var}(S_T) = S_0^2 e^{2\mu T} (e^{\sigma^2 T} - 1)$$



The Expected Return

- The expected rate of return is

$$E[S_T] = S_0 e^{\mu T}$$

$$(1/T) \ln[E(S_T)/S_0] = \mu$$

- The expected continuously compounded rate of return

$$v = (1/T) E[\ln(S_T/S_0)] = (1/T) \ln[E(S_T)/S_0] - \sigma^2/2 = \mu - \sigma^2/2$$



The BSOPM Formula

$$C = SN(d_1) - Ke^{-rT}N(d_2)$$

where $N(d_i)$ = the cumulative standard normal distribution function, evaluated at d_i , and:

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$N(-d_i) = 1 - N(d_i)$$



Example Calculation of the BSOPM Value

- $S = \$92$
- $K = \$95$
- $T = 50$ days ($50/365$ year = 0.137 year)
- $r = 7\%$ (per annum)
- $\sigma = 35\%$ (per annum)

What is the value of the call?



Solving for the Call Price, I.

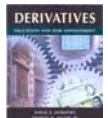
- Calculate the PV of the Strike Price:

$$Ke^{-rT} = 95e^{(-0.07)(50/365)} = (95)(0.9905) = \$94.093.$$

- Calculate d_1 and d_2 :

$$\begin{aligned}d_1 &= \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln(92/95) + (0.07 + 0.1225/2)0.137}{0.35\sqrt{0.137}} \\ &= \frac{\ln(0.96842) + 0.01798}{(0.35)(0.3701)} = \frac{-0.03209 + 0.01798}{0.12955} = -0.1089\end{aligned}$$

$$d_2 = d_1 - \sigma T^{.5} = -0.1089 - (0.35)(0.137)^{.5} = -0.2385$$



Solving for $N(d_1)$ and $N(d_2)$

- Choices:
 - Standard Normal Probability Tables (pp. 560-561)
 - $N(-0.1089) = 0.4566$; $N(-0.2385) = 0.4058$.
 - There are several approximations; e.g. the one in fn 8 (pg. 549) is accurate to 0.01 if $0 < d < 2.20$:

$$N(d) \sim 0.5 + (d)(4.4-d)/10$$

$$N(0.1089) \sim 0.5 + (0.1089)(4.4-0.1089)/10 = 0.5467$$

$$\text{Thus, } N(-0.1089) \sim \mathbf{0.4533}$$



Solving for the Call Value

$$\begin{aligned}C &= S N(d_1) - Ke^{-rT} N(d_2) \\ &= (92)(0.4566) - (94.0934)(0.4058) \\ &= \$3.8307.\end{aligned}$$

Applying Put-Call Parity, the put price is:

$$P = C - S + Ke^{-rT} = 3.8307 - 92 + 94.0934 = \$5.92.$$



Call Option (1)

- The value of the call option is

$$c = e^{-rT} E_{\rho} [\max(S_T - K, 0)]$$

- In a risk neutral world

$$\frac{\ln S_T - \ln S_0 - \left(r - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \sim \phi [0, 1]$$



Call Option (2)

- Need to know
 - Probability that $S_T > K$
 - Expected value of S_T given $S_T > K$

