KEELE UNIVERSITY

DEGREE EXAMINATIONS, 2007

P3/T3 (PRINCIPAL COURSE)

FRIDAY 27th APRIL 2007, 14.30-16.30

FINANCE
MANAGEMENT SCIENCE
BUSINESS ECONOMICS

ECO-30004

OPTIONS AND FUTURES

Candidates should attempt **all** questions from Section A (**30 marks**), **two** questions from Section B (**40 marks**) and **one** question from Section C (**30 marks**).

The use of hand-held, battery-operated, electronic calculators will be permitted subject to the regulations governing their use which are displayed outside the examination room.

The type of calculator must be specified on the cover sheet of your answer book.

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SECTION A

*Candidates should attempt to answer **ALL** questions from this section (30 marks)*

On 12th February 2007 the price of Mitchells & Butler stock was 760. The settlement prices of put and call options on this underlying on that day for some selected strike prices are reported in the following table:

Mitchells & Butlers plc				12 February 2007				
	700		720		760		800	
	C	P	**C**	P	**C**	P	**C**	P
March	**68.5**	5	**53**	8.5	**26.5**	23	**10.5**	49
June	**89**	19	**70**	25	**52**	42.5	**35**	66
Sept	**107.5**	29	**94**	35	**71.5**	53	**52**	74

1. On 12th February 2007, you buy a March put with a strike price of 800.

   (*a*) What is the intrinsic value of this option? What is its time value? **[4 marks]**

   (*b*) On the option's maturity date (19th March 2007), the price of Mitchells & Butler stock was 789. Explain how much you have gained or lost (ignore interest carrying costs). **[3 marks]**

2. For a given strike price, explain why the prices of the puts and calls rise with the length of time until maturity. **[5 marks]**

3. For a given maturity date, explain why the prices of the calls fall with a rise in the strike price. **[5 marks]**

4. Suppose that on 12th February 2007, you had decided to enter into a bear spread. You buy a June put on the stock with a strike price of 800 and write a June put on the stock with a strike price of 700.

   (*a*) Carefully explain the payoff you have on 12th February and the payoff you have at maturity in June. **[6 marks]**

   (*b*) What does your answer to part (*a*) imply about the relationship between the two put prices? **[2 marks]**

5. Consider the June call and put with a strike price of 700. Suppose the relevant interest rate, r, between 12th February 2007 and the expiry date in June is 1%. Show that $S_0 - K/(1+r) < C - P$ where S_0 is the original stock price, K is the strike price and C and P are the price of the call and put options. Identify an arbitrage opportunity? **[5 marks]**

SECTION B

*Candidates should attempt to answer **TWO** questions from this section (20 marks each)*

6. Adopting standard notation the Black-Scholes formula for the price of a European put option is

$$p = e^{-rT}\left(KN(-d_2) - S_0 e^{rT} N(-d_1)\right)$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and $N(x)$ is the cumulative probability distribution for a standard normally distributed variable. The derivative of $N(x)$ is

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

(*a*) Explain each of the terms in the Black-Scholes formula and give a brief intuitive explanation of the formula. **[15 marks]**

(*b*) Explain what happens to the put price as $\sigma \to 0$. **[5 marks]**

7. ABC stock either rises by 80% or falls by 20% with equal probability. The current price of ABC is 100 and the one-period risk free rate of return is 5%. The expected one-period rate of return on the market portfolio is 15%.

(*a*) What is the beta of ABC? What does beta measure? **[5 marks]**

(*b*) Consider a call option with a strike price of 100 that expires after one period. Calculate the risk-neutral probabilities and hence price the call option. **[5 marks]**

(*c*) Calculate the call option beta and the elasticity of the call option. Briefly explain why the call option elasticity is larger than one. **[5 marks]**

(*d*) Give a brief intuitive explanation of risk-neutral pricing and briefly explain why the risk-neutral probability for the "up-state" is lower than the true probability. **[5 marks]**

8. DCE stock sells at £150 per share. It will pay a dividend of £20 in six months time. The forward price for delivery of the stock in twelve months is F. The risk-free rate of interest over the twelve-month period is 10%.

(*a*) Calculate the theoretical forward price that allows no arbitrage profits to be made (use the simple interest rate for the six-month interest rate).
[5 marks]

(*b*) Suppose the actual forward price is F = £159. Give a full description of the cash and carry strategy that generates an arbitrage profit. How much profit is made?
[10 marks]

(*c*) Calculate the implied repo-rate on the cash and carry transaction. Is the implied repo-rate above or below the risk-free rate? Explain why.
[5 marks]

SECTION C

Candidates should attempt to answer ***ONE*** *question from this section. (30 marks)*

9. (*a*) Explain the relationship between the arithmetic mean and the geometric mean rate of return. **[10 marks]**

(*b*) Consider the stochastic differential equation

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dz(t) .$$

Explain the term $dz(t)$ in this equation. Why is this stochastic differential equation often used as a model of stock prices behaviour? **[10 marks]**

(*c*) Use Ito's lemma (see Appendix I) to find the stochastic process followed by $\ln(S)$ when S follows the process given in part (*b*). Hence find the distribution for $\ln(S_T)$. **[10 marks]**

10. Stock FGH either rises by a factor 4/3 or falls by a factor 3/4 in each of the next four periods. The current value of the stock is 144. There is a call option on the stock with a strike price of 119 which expires after four periods. The per-period interest rate is a constant 1/24. The prices of FGH at each node of the binomial tree, the values of the call option at each node and the dynamic delta hedging strategy are depicted in diagrams 1-3 in Appendix II.

(*a*) Explain why the stock price after four periods is asymmetrically distributed. **[5 marks]**

(*b*) Explain why at some nodes the delta hedging strategy involves holding no stock and at others it involves holding one unit of the stock. Explain how much is borrowed in these circumstances? **[5 marks]**

(*c*) Consider the date t=0 hedging strategy. Show that this replicates the call option value at date t=1. **[5 marks]**

(*d*) Consider the changes in stock holdings and borrowing that are required by the hedging strategy at the two nodes at date t=1. For each node, show that these changes are self-financing (ignore small rounding errors). **[5 marks]**

(*e*) Suppose the call is initially overpriced and trades at 60 instead of 54.8948. Explain how you can exploit this arbitrage opportunity. **[5 marks]**

(*f*) Briefly explain how the binomial model can be used to approximate the Black-Scholes continuous time model for option prices. **[5 marks]**

END

APPENDIX I - ITO'S LEMMA

Ito's lemma shows that for any process of the form

$$dx = a(x,t)dt + b(x,t)dz$$

then the function $G(x,t)$ follows the process

$$dG = \left(\frac{\partial G}{\partial x} a(x,t) + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2(x,t) \right) dt + \frac{\partial G}{\partial x} b(x,t) dz.$$

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Appendix II: Binomial Diagrams

Diagram I: Stock Price

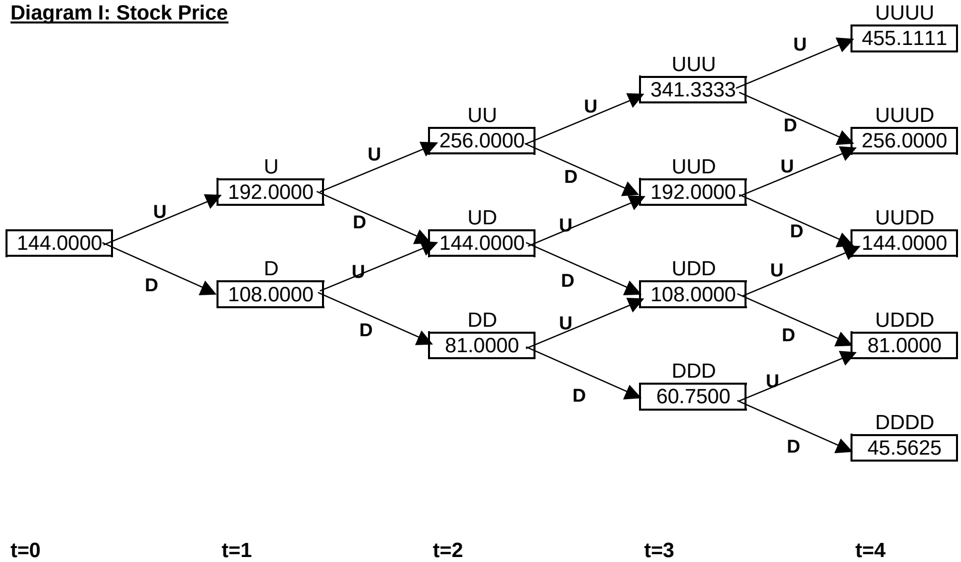

Diagram II: Call Value

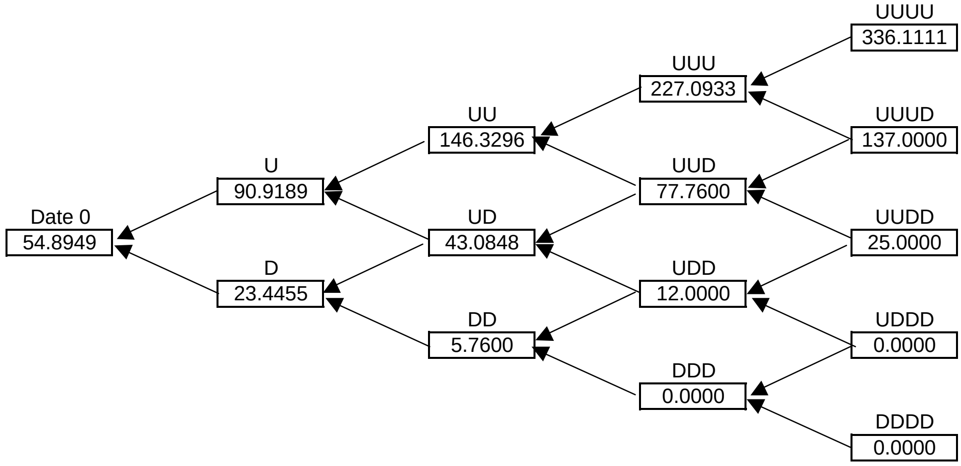

Diagram III: "Delta" and "B"

"Delta": top cell (hold S)
"B": Underneath (borrow cash)