

KEELE UNIVERSITY

ECO-30004

OPTIONS AND FUTURES

Candidates should attempt **all** questions from Section A (**40 marks**), and **two** questions from Section B (**30 marks each**).

MOCK EXAM

/Cont'd

## OPTIONS AND FUTURES

Candidates should attempt **all** questions from Section A (**40 marks**), and **two** questions from Section B (**30 marks each**).

### SECTION A

*Candidates should attempt to answer ALL questions from this section (40 marks)*

1. Let's say that  $F(0,T)=\$31$  per barrel and  $S(T)=\$32.20$  per barrel. The forward contract covers 24000 barrels of oil. Which party, the one that bought the contract, or the one that sold the contract, made the profit? How much is the profit? **[5 marks]**
2. In February 2008 you buy a put option on ABC stock with a strike price of 100 and expiry date in March 2008. The price of this option is 12. At the expiry date in March 2008, ABC stock trades at 85. What gain or loss have you made? (Ignore interest carrying costs). **[5 marks]**
3. A US mutual fund has invested in a portfolio of British stocks. However it does not wish to be exposed to exchange rate risk. Is the fund exposed to the risk that the  $\$/\pounds$  rate will rise or fall? To hedge, explain whether the fund should buy or sell British pounds forward? **[5 marks]**
4. A September call option with a strike price of 80 on DEF company has a current price of 12. If you write the call option, sketch the profit diagram at maturity. (Ignore interest carrying costs). **[5 marks]**
5. Briefly explain the similarities and differences between call and put options. **[5 marks]**
6. Briefly explain the similarities and differences between forward and futures contracts. **[5 marks]**
7. For a given maturity date, briefly explain why the prices of puts rise with a rise in the strike price. **[5 marks]**
8. What is a 2x5 FRA? Explain whether it is the buyer or the seller of an FRA who profits if the interest rate rises. **[5 marks]**

## SECTION B

Candidates should attempt to answer **TWO** questions from this section (30 marks each)

9. Adopting standard notation the Black-Scholes option pricing formula for the price of a European call option on a non-dividend paying stock is

$$c = S_0 N(d_1) - e^{-rT} KN(d_2)$$

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where  $N(x)$  is the cumulative probability distribution for a standard normally distributed variable.

- (a) Outline the assumptions behind the Black-Scholes formula. **[7 marks]**
- (b) Explain each of the terms in the Black-Scholes option pricing formula and give a brief intuitive explanation of the formula. **[15 marks]**
- (c) Use the put call parity condition to derive the Black-Scholes formula for the equivalent European put option. **[8 marks]**
10. Suppose the current spot £/\$ exchange rate is 0.5 and the one year forward rate is 0.6. Further suppose that the UK interest rate over the year is 1/5 and the US interest rate is 1/10.
- (a) If a UK bond with a face value of £60,000 with maturity in one year is available and has an interest rate of 1/5, what is the current price of the bond? **[6 marks]**
- (b) Suppose there is a US bond with a face value of \$100,000 with maturity in one year and an interest rate of 1/10. Outline all the relevant cash flows if you buy the US bond at the spot £/\$ exchange rate and sell \$ forward equal to the face value of the bond. **[6 marks]**
- (c) Calculate the implied repo rate on the transaction carried out in part (b). **[6 marks]**
- (d) How can the arbitrage opportunity be exploited? Explain what must happen to the forward rate if there is to be no arbitrage opportunity. **[6 marks]**
- (e) Suppose  $S$  is the current exchange rate,  $r$  the domestic interest rate and  $r^*$  the foreign interest rate. What is the theoretical forward price? Explain your answer. **[6 marks]**

11. Consider the binomial model where the stock either increases by a factor of 2 or stays the same (i.e.,  $U = 2$ ,  $D = 1$ ). The stock price is initially 100. The risk-free interest rate is  $\frac{1}{2}$
- Consider a put option with a strike price of 160 and expiry in one period. Calculate the  $\Delta$ -hedge needed to create a risk-less portfolio of the stock and the put option. **[10 marks]**
  - Calculate the risk neutral probabilities and use these probabilities to price the put option. **[5 marks]**
  - Now suppose that the put option expires in two periods (the strike price is still 160 and the initial value of the stock 100). Calculate the value of the put option at each node of the binomial tree when the option is of the European type. **[10 marks]**
  - Explain if the answer to part (c) changes if the option is of the American type. Calculate the prices of the American put at each node of the tree. **[5 marks]**
12. A Treasury bond with a face value of £1,000 and coupon rate of £50 every six months expires in twelve months time. The six-month simple interest rate is 7.18% per annum, the nine-month simple interest rate is 7.66% per annum and the twelve month simple interest rate is 7.90% per annum. Suppose there is a nine-month forward contract written on the twelve month Treasury bond. Let  $F$  be the forward price for this contract and let  $B$  be the price of the Treasury bond after nine months.
- Calculate the current price of the Treasury bond. **[6 marks]**
  - Calculate all the cash flows for a period of up to nine months from today, for the following strategy: buy the treasury bond today and borrow risk-free for six months the present value of the first coupon payment of the bond. **[8 marks]**
  - Calculate all the cash flows for a period of up to nine months from today, for the following strategy: go long in the forward contract and invest the present value of the forward price for nine months in the risk free asset. **[8 marks]**
  - Use your answers to parts (b) and (c) to determine the forward price  $F$ . Explain how you would exploit an arbitrage opportunity if the actual forward price was below the price you have calculated. **[8 marks]**