Eco-30004 Options and Futures
Mock Exam 2008 – Outline Solutions

Question 1

The buyer of the forward contract makes a profit of \$31.20-31=\$1.20. The total profit per contract is \$1.20(24000)=\$28,800.

Question 2

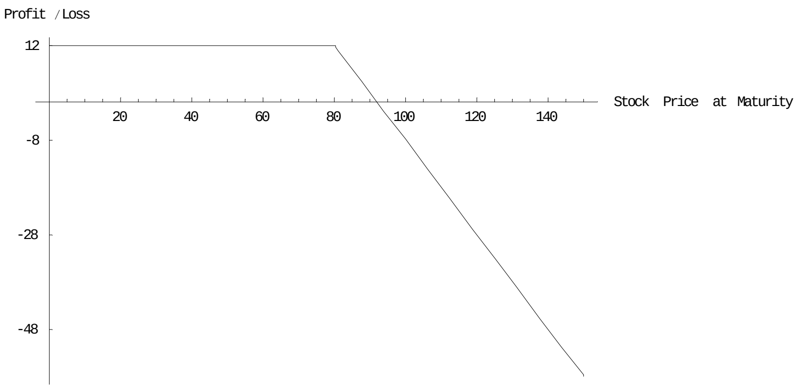
The put option can be exercised for a profit of 100-85=15 at maturity. Since the option cost 12 and we ignore interest carrying costs, the net profit is 3 per option.

Question 3

The fund is exposed to the risk that the \$/£ exchange rate will decline. The fund has GBP denominated assets, but no GBP-denominated liabilities. GBP should be sold forward to hedge.

Question 4

Writing a call option gives a payoff of $-(S_T-K)$ at maturity if $S_T>K$ and zero otherwise where S_T is the price of the stock at maturity and K is the strike price. We are given K=80 and initial price of 12.



Question 5

Puts give the right to sell, calls give the right to buy. The call pays off if the stock rises above the strike price, the put if it falls below it. For more details see the options notes.

Question 6

Forwards and futures both give the obligation to buy at a given delivery price at a specified date in the future. Futures are exchange traded and marked to the market every day. Forwards are not marked to the market. Futures and forwards prices are identical if the interest rate is known.

Question 7

The put gives the right to sell, so the right to sell at a higher price will be worth more. The payoff from the higher strike price put always dominates and is therefore worth more. See the option notes for more details.

Question 8

A 2x5 FRA is a forward rate agreement in which the buyer agrees to borrow for a three month period in two months time at a fixed interest rate. If the interest rate rises above the rate agreed then the buyer gains as the borrower prefers to borrow at the lower fixed rate (see pages 56-59 in the textbook).

Question 9

Largely bookwork. See textbook and notes. Part c) uses the symmetry of the normal distribution: $N(x)=1-N(-x)$.

Question 10

a) The current price of the bond is the discounted value: £60,000/(1+0.2) = £50,000.
b) The face value of the bond is \$100,000 so the current price is \$100,000/(1+0.1) = \$1,000,000/11 At the current exchange rate the cost of the bond is 0.5 times this or £500,000/11 = £45454.54. Selling dollars to the face value of the bond forward will deliver 0.6 times \$100,000 or £60,000.
c) The implied repo-rate is given by (60,000-45454.54)/(45454.54)=8/25=0.32 or 32%.
d) There is an arbitrage opportunity as the long-trip return is higher 32% than the lending directly (20%).
   Today: We borrow £50,000 domestically today (equivalently sell the UK bond) and use this to buy the US bond at the current exchange rate leaving \$50000-0.5(45454.54) = £4545.45 profit. We sell \$100,000 forward at the 0.6 forward rate.
   In one year: The US bond yields \$100000 which is converted to £s at the forward rate to give £60,000. This is exactly enough to repay the UK loan (face value of the UK bond) given the 20% domestic interest rate. Thus we have an arbitrage profit.
e) By analogy with the example the rate of return on the long and short trip must be equal. Investing 1 unit domestically yields $1+r$ after one year. For the long trip 1 unit converted at the current exchange rate S gives $(1/S)$ units of the foreign currency which invested at the foreign interest rate yields $(1/S)(1+r^*)$ units of the foreign currency at the end of one year. Converting this back at the forward rate F gives $(F/S)(1+r^*)$ units for the one unit of domestic currency invested. Hence in the absence of arbitrage $F=S(1+r)/(1+r^*)$.

Question 11

a) The end stock prices after one period are 100 (down state) and 200 (up state). The put is worth 160-100=60 in the down state and 0 in the upstate. If we buy Δ units of the stock and the put the value of the portfolio is 200Δ in the up state and $100\Delta + 60$ in the down state. This is risk-less if $200\Delta=100\Delta+60$ or $\Delta=3/5$. The value of the portfolio at the end of the period is 200(3/5)=100(3/5)+60=120. As the interest rate is 50%, the value today is 120/(1+1/2)=80. Since 3/5 units of the stock cost 60 today, the remaining value of 20 is the price of the put.

b) The risk-neutral probabilities satisfy the price = expected discounted return equation. For the asset this gives 100=(200p+100(1-p))/(1+1/2) or 100p+100=150. Hence p=1/2. For the put option the discounted expected value is (p0+(1-p)60)/(1+1/2) = 40(1-p) and with p=1/2, the put price is 20 as in part a).

c) The ending values after two periods are:
400 (two ups); 200 (one up one down); 100 (two downs).
Thus the put is worth 0 (two ups); 0 (one up one down), 60 (two downs).
Using the risk-neutral probabilities, the value of the put after the first up is 0 (it is worth 0 whatever happens thereafter) and (p0+(1-p)60)/(1+1/2) = 40(1-p)=20 after one down.
Hence the initial value is (p0+(1-p)20)/(1+1/2) = (40/3)(1/2)=20/3.

d) If the option is American it can be cashed in early after the first down for a profit of 60. Likewise in the initial period the American option must be worth exactly 60 as it can be exercised for an immediate profit of 160-100=60.

Question 12

a) The price of the bond is the discounted value of the future payments:
price of bond = £50/[1+0.0718(1/2)] + £1050/[1+0.079] = £48.2672+£973.123 = £1021.39.

b) Buy Treasury bond at cost of £1021.39 and borrow present value of coupon payment (£48.2672) for a net cost of £973.123. The cash flow after six months is zero as the coupon on the bond repays the loan. After nine months we own a treasury bond worth B.

c) Today we buy the forward contract at price F and invest the discounted value of the forward price, that is F/[1+0.0766(3/4)]. There are no cash flows after six months. After nine months the forward contract yields a profit or loss of F-B and the investment yields a return of F for a net value of B.

d) The strategies yield the same payoff after nine months, namely B, so must in the absence of arbitrage cost the same to put together. That is
F/[1+0.0766(3/4)]= £1050/[1+0.079] or
F=£1050[1+0.0766(3/4)]/[1+0.079]=£1029.03.