Class Assignment

The class assignment examines the binomial option pricing model. You will be required to compute the value of a call option in a four-period binomial tree. Some calculation is involved but this can be done either on a calculator or in a spreadsheet (or indeed by hand). Marks are awarded for correct computation but most marks will be awarded for correct and full explanations. Therefore you should give complete explanations for your answers. You will find help in answering the question both during the course, in the course text and from other textbooks available in the library. You are expected to seek this information out to help you with your assignment.

Hand-in: The hand in date is 22nd April between 10.00 a.m. and 13.00 p.m. at the school postgraduate office in the Darwin Building.

Assessment: This assignment is worth 30% of the module mark. The pass mark for the assignment is 50%.

The Traded Assets

Underlying Asset: Stock DOB is currently traded at a price of S_0 where S_0 is the your year of birth. Thus if you were born in 1980, the initial stock price is $S_0 = 80$. During any of the next four periods, the price of the stock will either go up by factor $U = \frac{4}{3}$ or down by factor $D = \frac{1}{U} = \frac{3}{4}$.

Risk-free Bond: For all periods, the risk-free one-period interest rate is $r = \frac{1}{24}$. Thus it is possible in every period to trade a risk-free bond (or fractions or multiples of that bond) that delivers 100 next period at a current price of 96.

Call Option: There is a European call option with a maturity in four periods which has a strike price of $K = 90$.

Answer the Following Questions

(a) Draw a binomial tree indicating the stock price at every node of the tree.

(b) Calculate the risk-neutral probabilities and the risk-neutral probability of reaching each node of the tree. What can be said about the relationship between the risk-neutral probabilities and the actual probabilities?

(c) Calculate the value of the call option at every node of the tree. At what nodes is the call option in-the-money? At what nodes is it out-of-the-money?

(d) Find the minimum number of "up branches", k, that must be taken such that the price of DOB stock at the terminal node S_T is greater than the strike price K. Let $B_a(k)$ denote the probability that the binomial random variable with parameter a is greater than k after 4 periods. This is given by

$$B_a(k) = \sum_{i=k}^{4} \binom{4}{i} a^i (1-i)^{4-i}.$$

Show that the call option price at the initial date satisfies

$$C_0 = S_0 B_s(k) - \frac{K}{(1+r)^4} B_p(k)$$

where

$$p = \frac{R-D}{U-D}, \qquad s = \frac{pU}{R},$$

and $R = 1 + r$. Give an intuitive interpretation for this formula.

(e) Find a trading strategy that replicates the call by a levered long stock: "buy Δ shares of the stock, borrow m pounds." Draw a tree diagram to show what happens to Δ and m at each node of the tree. Given an explanation for the changes in Δ and m in the tree.

(f) Demonstrate that the delta-hedging strategy is self-financing.

(g) Suppose the call option was overpriced at the initial node by an amount X. Show that there is a risk-less arbitrage opportunity to be had by writing the call. Explain why the fact that the delta-hedging strategy is self-financing is important for your result.

(h) Suppose that you try to exploit this arbitrage opportunity but at date 3 you are unable to liquidate any of the assets you hold. How does this affect your profits? Discuss.

(i) Explain how the binomial option pricing model can be extended to many periods so as to approximate the Black-Scholes pricing formula for a call option

$$C_0 = S_0 N(d_1) - K e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\log(\frac{S}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T},$$

$N(d)$ is the cumulative standard normal distribution and σ is the standard deviation of the returns on the underlying stock. Give an intuitive explanation for this formula.