FIN-40008 FINANCIAL INSTRUMENTS

EXERCISES

1. Suppose you buy a call option at price C on a underlying asset with current price S_0. The option expires at date T and has a strike price of K.

   (a) What is the intrinsic and the time value of the option?

   (b) Describe the payoff you have at the expiration date T.

   (c) Explain why it can be expected that $C \leq S_0$.

2. For a given strike price, briefly explain why the prices of the American puts and calls rise with the length of time until maturity.

3. Consider two European put options on stock ABC with strike prices 10 and 20 and the same maturity date T. The prices of the two options today (at date $t < T$) are $p(10)$ and $p(20)$ respectively. Briefly explain why $p(20) > p(10)$. Consider a **bear spread** with puts by writing the put option with the strike price of 10 and buying the put with the strike price of 20. Detail the cash flows now at date t and the cash flows at the maturity date T. Explain why there would be an arbitrage opportunity if $p(10) > p(20)$.

4. Consider the one period binomial model with parameters $U = 2$ and $D = 1/2$ (i.e., where the stock increases by a factor of 2 or decreases by a factor of 1/2). Suppose that the stock price is initially 150 and that the risk-free interest rate is 50% per period.

   (a) Consider a call option with a strike price of 165 and expiry in one period. Construct the Δ-hedge needed to create a risk-less portfolio of the stock and a written call. Verify that the payoffs to the strategy are risk-free.

   (b) Calculate the cost of the portfolio in part (a) and hence value the call option.

5. Derive the put-call parity condition $C - P = S_0 - Ke^{-rT}$ for European options on a non-dividend paying stock. If $C - P > S_0 - Ke^{-rT}$ what portfolio would you hold to exploit the arbitrage opportunity?

6. Consider two European call options on an underlying stock with strike prices K_1 and K_2 where $K_2 > K_1$. Both options expire on the same date. Let the price of these two call options be C_{K_1} and C_{K_2}. Suppose you form a bull spread by buying the option with the lower strike price K_1 and selling the call option with the higher strike price K_2.

   (a) Describe the payoff you have at the expiration date.

   (b) Using your answer to part (a) explain why $C_{K_1} > C_{K_2}$.

   (c) Now consider borrowing the discounted value of the difference in the strike prices in addition to the bull spread. What does the payoff to this new portfolio imply about the relationship between the prices C_{K_1} and C_{K_2}?

7. There is a one-year forward contract on an asset that provides no income over the year. Storage costs are 2 per unit and are paid at the end of the year. The current spot price is 450, the price of a forward contract is 500 and the risk-free rate of interest over the year is 7%.

   (a) Show there is an arbitrage opportunity by selling the forward contract. Carefully explain all cash flows now and at maturity that lead to an arbitrage profit.

   (b) Calculate the theoretical forward price in the absence of any arbitrage opportunities.

8. Consider a one period binomial model. The stock price is initially 100 and rises to either 120 or 180 (in this case $u = 0.8$ and $d = 0.2$). Suppose that the rate of interest is 40% and that both the changes in price are equally likely (that is, $r = 0.4$ and $\pi = 1/2$). Suppose there is a put option with a strike price of 141 that expires after one period.

   (a) Calculate the expected return on the stock, the volatility (standard deviation) and the risk premium.

   (b) How can you create a synthetic put option with a strike price of 141 using only the underlying stock and the risk-free asset?

(c) Use the synthetic portfolio to determine the price of the put option.

(d) Calculate the risk-neutral probability in this example. Use the risk-neutral probability to price the put option.

(e) Calculate the value of Δ required to create a riskless portfolio of the stock and the put option. How can the put option be priced using this approach?

(f) Compute the risk premium on the put option. Calculate the standard deviation of the put return.

(g) Calculate the put option elasticity.

(h) If the return on the market portfolio is 60%, calculate the *beta* of the underlying stock. what is the *beta* of the put option?

(i) Calculate the ratio of the risk premium of the put option to the risk premium on the underlying stock. Calculate the ratio of the βs. Calculate the ratio of the volatilities.

9. Consider a two-period binomial model with $u = 3/10$, $d = -1/5$ and $r = 1/20$. The initial stock price is 100.

   (a) Calculate the stock price at each node of the tree.

   (b) Calculate the risk-neutral probability.

   (c) Consider a put option with a strike price of 99 that expires in two periods. Calculate the put option value at all nodes of the tree if the option is of the European type.

   (d) Find the Δ-hedging strategy that replicates the put option values.

   (e) Consider the nodes of the tree after one period. Show that the Δ-hedging strategy is *self-financing*.

   (f) How do the put values change if the option is of the American type?