Exotic Options

These notes describe the payoffs to some of the so-called *exotic* options. There are a variety of different types of exotic options. Some of these options are *path dependent* so that the value of the option depends on the previous values of the underlying as well as the current price of the underlying.

Keywords: Digital options, Barrier options, Path dependent options, Lookback options, Asian options.

Reading Hull Chapter 22 [Just read about the different types exotic options and their payoffs. Ignore the equations describing how they are priced for the moment.]

Introduction

American and European puts and calls are sometimes referred to as plain vanilla options. There are however a wide variety of other types of options that are traded. These are sometimes referred to as **exotic options**. In the past most exotic options were traded in over the counter markets rather than exchanges, but more recently these alternative option types can be found on exchange markets too. The variety and types of options that are available are really only limited by imagination. As long as the payoff can be well specified, it can depend on any property of the price of the underlying asset. The following list just gives a few examples.

Digital Options

One simple type is the digital or binary option. The payoff to a digital option depends on whether the price of the underlying asset at maturity is greater

or less than the strike price. Thus for a digital call option the payoff at maturity is:

$$c_T^b = \begin{cases} 0 & \text{if } S_T \leq X \\ 1 & \text{if } S_T > X \end{cases}$$

and the payoff at maturity to a digital put option is:

$$p_T^b = \begin{cases} 1 & \text{if } S_T \leq X \\ 0 & \text{if } S_T > X. \end{cases}$$

Range Forward Contract

A range forward contract specifies a band $[X_2, X_1]$. If the price of the underlying at maturity is within the band then the holder is obliged to buy the asset at its current price. If the price of the underlying at maturity is below X_2 then the holder has to buy at X_2 and if the price of the underlying at maturity is above X_1 then the holder has to buy at X_1. The range forward is designed so that the initial value of the contract is zero and therefore similar to a forward contract. It is equivalent to a combination of a long call option with a strike price of X_1 and a written put option with a strike price of X_2, where the strike prices are chosen so that the put and the call have the same prices. The payoff at maturity to a range forward contract is $\max[S_T - X_1, 0] - \max[X_2 - S_T, 0]$.

Break Forward

A break forward contract specifies a delivery price K and a break price B at which the forward price can be broken. Thus at the maturity date, the break price will be paid on a long forward position if $-B > S_T - K$. For example if the break price is \$10 and the delivery price is \$100, the payoff to the long forward position is $S_T - 100$. If $S_T < 90$, there will be a loss of more than \$10 and it will be worth while to pay the break price of \$10. The payoff

at maturity is therefore $\max[S_T - 100, -10] = \max[S_T - 90, 0] - 10$. This then is just like a call option with a strike price of $90 but where a payment of $10 is paid at maturity. Since the break forward costs nothing when it is initiated, it is essentially a deferred payment option, where the payment of the initial cost of the option is deferred until maturity. Suppose that the interest rate until maturity is $\frac{1}{9}$, then in our example a call option with a strike price of $90 which costs $9 us equivalent to a long break forward with a delivery price of $100 and break price of $10. In general letting r denote the interest to maturity and c_t the current price of the call option with a strike price X, then the call option is equivalent to a long position in a break forward contract with delivery price $X + (1 + r)c_t$ and break price $(1 + r)c_t$.

Rainbow Options

It is possible to write options on more than one underlying asset. Such options are called rainbow options. For example a put option may specify that you have the option to deliver one from a range of different assets. Clearly if the exercise price is the same for all assets specified, and if you decide to exercise your option to sell, you will choose to deliver that asset with the lowest current price. Thus if there are two assets specified and their prices are S^1 and S^2, then the intrinsic value of the put option is

$$x\text{Intrinsic value} = \begin{cases} X - \min[S^1, S^2] & \text{if } \min[S^1, S^2] \leq X \\ 0 & \text{if } \min[S^1, S^2] > X. \end{cases}$$

"As-you-like-it" Options

This is an option where the holder can decide at a specific time whether the option is a put or a call option. Suppose that the where the decision must be made is t. The value of the option at this time is $\max[c_t, p_t]$. If the as-you-like-option offers the choice between a European put and a European

call with the same strike price and same maturity date T, then the put call parity condition can be used and

$$\max[c_t, p_t] = \max\left[c_t, c_t + \frac{X}{(1+r)} - S_t\right] = c_t + \max\left[0, \frac{X}{(1+r)} - S_t\right]$$

where r is the risk-free interest rate between t and the maturity date T. The as-you-like-it option therefore consist of a portfolio of a call option with strike price X and maturity at date T and a put option with strike price $\frac{X}{(1+r)}$ and maturity at date t.

Barrier Options

Barrier options have a payoff that depends on whether the underlying asset reaches a certain level, the barrier, prior to maturity. There are two main varieties of barrier option. The **knock-in** only pays out if the price of the underlying reaches the barrier and the **knock-out** only pays out if the underlying does *not* reach the barrier. These can be further classified by whether the barrier is set above or below the initial value of the underlying asset. If the barrier is above the initial value of the underlying, it is said to be an **up** option. If the barrier is below the initial value of the underlying asset, it is said to be a **down** option. The payoff at maturity for a **down-and-out call** option is

$$c_T^{down-out} = \begin{cases} c_T & \text{if } S_t > B \quad \text{for all } t \leq T \\ 0 & \text{if } S_t < B \quad \text{for any } t \leq T \end{cases}$$

where B is the barrier and c_T is the value of the plain vanilla call option. Clearly if you own both a down and out call option together with a down an in call option on the same underlying with the same barrier, strike prices and maturity, then you have a plain vanilla call option. So for calls and puts and down and up options:

$$\text{vanilla} = \text{in} + \text{out}.$$

Barrier option sometimes specify a double barrier with an upper and lower limit. Sometimes the barrier changes over time. Sometimes a rebate is paid if the barrier is hit. Some options become barrier options if a particular value, usually the strike price, for the underlying is reached.

Lookback Options

Lookback options have a payoff that depends on the maximum or minimum value that the underlying asset reaches during the lifetime of the option. The payoff to maturity of a European-style lookback call is $S_T - S_{min}$ where S_{min} is the minimum value that the underlying achieves over the option's lifetime. The payoff to maturity of a European-style lookback put is $S_{max} - S_T$ where S_{max} is the maximum value that the underlying achieves over the option's lifetime. Note that if you hold both a lookback put and a lookback call your payoff is $S_{max} - S_{min}$. It is like you bought when the price was lowest and sold when the price was highest. Of course such sweet deals are likely to be costly.

Asian Options

An Asian option has a payoff that depends on the average price of the underlying asset from its starting date. Thus the intrinsic value of an average price Asian call option is $\max[0, S_{ave} - X]$, where S_{ave} is the average value of the asset from the start of the option to the current date. Another type of Asian option is the average strike price type, where the strike price is the average value. In this case the intrinsic value of the call is $\max[0, S_t - S_{ave}]$ and the intrinsic value of the put is $\max[S_{ave} - S_t, 0]$.