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Debt with Potential Repudiation:  
Short-Run and Long-Run Contracts

by Tim Worrall\*

UNIVERSITY OF READING  
DEPARTMENT OF ECONOMICS

Department of Economics,  
University of Reading,  
P.O.Box 218  
Whiteknights,  
Reading, R66 2AA  
UK

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ABSTRACT

Lending across national boundaries is different from lending within national boundaries for two reasons. First the lack of legal sanction if debt is repudiated and second the difficulty of enforcing collateral requirements. This paper examines a model of international lending where the borrower is allowed to repudiate if this is to his advantage. There is an infinite time horizon and the borrower has an i.i.d. income stream. We consider two cases: first where the lender must break-even period by period and second where the lender only needs to break-even in the long-run. It is shown that the two cases are equivalent and that, although debt is initially restricted, consumption is completely stabilised in the long-run.

## INTRODUCTION

Lending across national boundaries is different from lending within national boundaries because the borrower's assets cannot, except in exceptional circumstances, be sequestered. Without possible sequestration collateral requirements cannot be enforced and there is no legal sanction against repudiation of debt by the borrower. The borrower will repudiate if he believes he can gain an advantage. In the words of Eaton and Taylor (1986) the borrower will repudiate if he is insouciant as well as if he is illiquid or insolvent.

There are a number of possible non-legal sanctions against repudiation; trade embargos, diplomatic and political pressure, withdrawal of trade finance and withdrawal of future credit. This paper, like Eaton and Gersovitz (1981) concentrates on the withdrawal of future credit. In particular it is assumed that any default is universally observable and that banks will refuse to provide any future credit once a lender has defaulted.

There is strong evidence from North Korea and Rhodesia that trade sanctions do not work. Though whether withdrawal of future credit is any more effective is debatable. However much of the analysis of the paper would go through if withdrawal of credit was for a finite period rather than forever.

## THE MODEL

There are two types of agents: sovereign borrowers and international banks. The borrower has a per-period strictly concave utility (or objective) function,  $u$  defined on consumption (or absorption),  $c$ . It is assumed  $u$  is twice continuously differentiable and  $\lim_{c \rightarrow 0} u'(c) = -\infty$ . The borrower's income,  $y$  is i.i.d. over an infinite number of dates,  $t=0,1,\dots,\infty$ , and has one of  $S$  possible values,  $y_1 \leq y_2 \leq \dots \leq y_S$ , with probability  $p_s, \sum_{s \in S} p_s = 1$ .

Banks are risk neutral and like the borrower discount the future by a constant factor  $\alpha$ . They can observe the borrower's income at every date so the amount loaned and the debt-service payments incurred can be made conditional upon income. They cannot sequestrate the borrower's assets and the only sanction against default is the withdrawal of future credit. All banks are aware of any default and observe a moratorium on future lending.

## DEBT CONTRACTS

Suppose a bank agrees to a loan of  $b$  in the current period and expects a repayment  $d'_s$  next period if income is  $y_s$ . Such a loan is feasible if

$$b = \alpha E d'_s \quad (F)$$

where  $E$  is the expectation taken over income next period. So a loan is feasible if it equals the expected discounted debt-service obligations. The bank then breaks-even period by period. So (F) is the zero profit condition for one period loans. Clearly new loans will not be granted unless debt-service obligations have been met.

Let  $z_s = y_s - d'_s$  be the net income in state  $s$  and denote net income next period by  $z'$ . In the initial period  $z_s = y_s$  for each  $s=1,2,\dots,S$ . Consumption is net income plus new loans,  $c=z+b$ . Given  $z$  the borrower and banks will agree a loan and repayment schedule to maximise the borrower's utility. Let  $U(z)$  denote the maximum future expected discounted utility of the borrower taking into account all future loans. If the borrower is not to default.

$$U(z_s) \geq u(y_s) + \alpha u(y_*) / (1-\alpha) \quad s=1,2, \dots, S \quad (ND)$$

where  $y_*$  is the certainty equivalent income, i.e.  $u(y_*) = E u(y_s)$ .

The no-default condition says the borrower must do as least as well by meeting his debt-service obligations as defaulting and going on his own thereafter.

The function  $U(z)$  is defined recursively by the principle of optimality.

The optimality equation is

$$U(z) = \max_{b, \{d'_s\}_{s \in S}} u(z+b) + \alpha E U(y_s - d'_s) \quad (O)$$

subject to (F) and (ND). Substituting condition (F) and letting  $\lambda_s$  be the multiplier for condition (ND) the first order conditions are

$$\alpha u'(z+b) = (\alpha + \lambda_s) U'(z'_s), \quad s=1, 2, \dots, S \quad (FO)$$

Together with the envelope condition

$$u'(z+b) = U'(z) \quad (E)$$

they determine the optimum loan and the borrower's consumption.

From (E),  $U(\cdot)$  is strictly concave so standard arguments imply the existence of a unique optimum. Also  $z$  is bounded away from zero or  $y_s \geq d'_s$  for every  $s=1, 2, \dots, S$ . So the borrower is never illiquid or insolvent.

If the borrower is completely honest and does not default even if it is to his advantage then  $z'_s = z$  in every state  $s=1, 2, \dots, S$ . Consumption will be completely stabilised at  $c = \alpha y^* + (1-\alpha)y_s(0)$  where  $y^* = E y_s$  and  $y_s(0)$  is the actual income at date  $t=0$ .

If however, the borrower would default the condition (ND) must be imposed. Then

RESULT 1 There is a critical state  $c$  such that  $z'_q$  is constant for  $q=1, 2, \dots, c$ , and for  $q=c+1, c+2, \dots, S$ ,  $U(z'_q) = u(y_q) + \alpha u(y_*) / (1-\alpha)$ .

PROOF Let  $U(z'_{c+1}) = u(y_{c+1}) + \alpha u(y_*) / (1-\alpha)$ . By (ND)  $U(z'_q) > U(z'_{c+1})$  for  $q > c+1$ . But then from (FO)  $\lambda_q > \lambda_{c+1} > 0$ . Therefore  $\lambda_q = 0$  for  $q < c+1$  and  $U'(z'_q) = U'(z+b)$  independent of  $q$ .

RESULT 2 Let  $c$  be the critical state at date  $t$  and suppose state  $s$  occurs. Let  $m = \max(c, s)$ . Then  $z'_q = z_s$  for  $q=1, 2, \dots, m$ ;  $z'_q = z_q$ ,  $q=m+1, m+2, \dots, S$ .

PROOF Consider  $q=1, 2, \dots, m$  and suppose  $\lambda_q > 0$ . Then  $z'_q \leq z_q \leq z_s$  by (ND) and Result 1. But from (FO)  $z_s < z'_q$ . So  $\lambda_q = 0$  and  $z'_q = z_s$ . Next consider  $q=m+1, m+2, \dots, s$  and suppose  $\lambda_q = 0$ . Then  $z'_q = z_s$  from (FO) and  $z_q > z_s$  by Result 1. But this implies  $z'_q > z'_q$  which violates (ND). So  $\lambda_q > 0$  and  $z'_q = z_q$ .

Result 1 shows the borrower is constrained by the non-default condition when income is 'high'. This is to be expected, 'high' income states are likely to be associated with net repayments. In 'low' income states net income and hence consumption is stabilised.

Result 2 shows how 'high' income states are defined. The critical state depends upon past history. If the critical state at  $t$  is  $c$  and state  $s$  occurs then the critical state at date  $t+1$  will be the greater of  $c$  or  $s$ . At date  $t=0$   $z_s = y_s$  and (ND) is satisfied so the critical is the lowest state such that  $y_s \leq z_{c+1}$  where  $z_{c+1}$  satisfies  $U(z_{c+1}) = u(y_{c+1}) + \alpha u(y_*) / (1-\alpha)$ .

COROLLARY Let  $c'_q$  be next periods consumption in state  $q$ . Then

$$c'_q = c_s \quad \text{for } q=1, 2, \dots, m \text{ and } c'_q = c_q, q=m+1, m+2, \dots, s.$$

PROOF From (O)  $b$  is a function of  $z$ . Let  $b_q = b(z_q)$ . Then  $c'_q = z'_q + b(z'_q)$ .

So for  $q \leq m$   $c'_q = z_s + b_s = c_s$  and for  $q \geq m+1$ ,  $c'_q = z_q + b_q = c_q$ .

Consumption is also non-decreasing in income.

RESULT 3  $0 \leq dc/dz \leq 1$

PROOF Using Result 2 in (O) and differentiating  $dc/dz = (1-\alpha) \sum_{q=1}^m p_q$ .

So consumption never decreases. If today's income is higher than yesterdays then consumption increases but by the smallest possible amount to prevent the borrower from defaulting. If today's income is less than yesterdays then consumption remains the same.



In the long run consumption will be perfectly stabilised.

Let  $c^*$  be the long run value of consumption and define  $z^* = \max(y_s(0), \underline{z}_S)$  where  $y_s(0)$  is income at date  $t=0$ .

PROPOSITION 1 In the long run  $c^* = \alpha y^* + (1-\alpha)z^*$ . There exist  $\alpha^*$  and  $\alpha_*$  such that for  $\alpha > \alpha^*$ ,  $z^* = y_s$ ,  $s=1, 2, \dots, S$ ; and for  $\alpha < \alpha_*$ ,  $z^* = \underline{z}_S$ .

PROOF Since state  $S$  must occur in finite time, by Result 2 the long run value of net income is  $z^*$ . Then by (F)  $c^* = \alpha y^* + (1-\alpha)z^*$ . Therefore  $u(\alpha y^* + (1-\alpha)\underline{z}_S) = \alpha u(y^*) + (1-\alpha)u(y_S)$ .

So for a sufficiently small  $\underline{z}_S \geq y_{S-1} \geq y_{S-2} \geq \dots \geq y_1$

And  $y_S - \underline{z}_S > \alpha(y^* - y_*) / (1-\alpha) > 0$ , so for  $\alpha$  sufficiently large

$\underline{z}_S \leq y_1 \leq y_2 \leq \dots \leq y_S$ .

For  $\alpha$  sufficiently large the no-default conditions (ND) have no impact. However for  $\alpha < \alpha^*$ , at least for some initial states, long run consumption will be higher than it would have been without the no-default constraints. Of course if  $\alpha$  is relatively low this long-run value is discounted quite a lot and in the short-run consumption will be lower. For  $\alpha < \alpha_*$ , long-run consumption is greater than  $y_{S-1}$ .

By way of an example consider the stochastic model of Eaton and Gersovitz (1981). Income is, with equal probability,  $1 + \sigma$  or  $1 - \sigma$ . Let  $p = y^* - y_*$  be the risk premium. If the borrower has constant absolute risk aversion then  $p = \frac{1}{2} \sigma^2 A$ , where  $A$  is the Arrow-Pratt coefficient of absolute risk aversion. Let  $q = \bar{y} - \alpha y^* - (1-\alpha)\underline{z}_2$ , where  $\bar{y} = (1-\alpha)y_2 + \alpha y_*$ . Then  $q = \frac{1}{2} \alpha (1-\alpha) (p + \sigma)^2 A$ , so that  $\underline{z}_2 = (1+\sigma) - (q + \alpha p) / (1-\alpha)$ . Differentiating,  $\underline{z}_2$  is decreasing in  $\alpha$  and  $A$  but maybe increasing or decreasing in  $\sigma$ . So the higher the discount factor and the higher the degree of risk aversion the less likely it is that the (ND) constraints will bind.

## LONG-RUN CONTRACTS

Condition (F) restricts loans to one period only. Debt-service payments must be made before any new loan can be granted. In this section it is shown that no long term contract can do any better.

A long-run loan contract is a sequence  $(a_t)_{t=0}^{\infty}$  where  $a_t$  is the net lending at date  $t$ . It will depend on the entire past history of income. Given a long-run contract let  $U_s$  be the expected discounted utility to the borrower from the contract from date  $t$  onwards. It is defined recursively by  $U_s = u(y_s + a_s) + \alpha EU'_q$ . If the borrower is not to default then  $U_q \geq u(y_q) + \alpha u(y_*) / (1 - \alpha)$ .

The bank will choose the contract which maximises its profits. By Bellman's Principle if the contract maximises profits at date  $t$  it does so from date  $t+1$ . Let  $d(U_s)$  be the bank's maximum profits at date  $t$  if the borrower gets  $U_s$ . It is defined recursively by

$$d(U_s) = \max_{a_s, (U'_q)_{q=1}^S} -a_s + \alpha E d(U'_q) \quad (0')$$

$$\text{s.t.} \quad u(y_s + a_s) + \alpha E U'_q = U_s$$

$$U'_q \geq u(y_q) + \alpha u(y_*) / (1 - \alpha) \quad q=1, 2, \dots, S$$

The two optimality equations (0) and (0') are equivalent. Net lending is  $a=b-d$ , so letting  $d_q = d(U'_q) = U(z'_q)$ , (0') implies  $d_s = \alpha E d'_q$  which is condition (F).

PROPOSITION 2 One period contracts and long-run contracts are equivalent.

## CONCLUSIONS

The model we have examined is very simple. Yet there are some strong conclusions about how consumption is stabilised and how one period loans are as good as long-run contracts.

There are, however, a number of interesting problems not taken into account. The lender is assumed to be able to observe the borrowers income, so there is symmetric information. A natural extension is to assume that there is asymmetric information and the bank cannot observe income. Also the bank is assumed to observe total indebtedness. This is probably unrealistic, for example in Nigeria even the federal government was unable to keep track of state borrowing.

Also no new information becomes available through time so there is no role for renegotiating or rescheduling of debt. Neither has the denomination of the debt been considered, everything is in real terms. But most debt is denominated in dollars to prevent sovereign debtors from increasing the inflation rate to service the debt. This causes problems when there is unexpected changes in interest rates or exchange rates.

Neither is there any role for capital. A useful extension might be along the lines of Kletzer (1984) who has foreign capital as a distinct factor of production in the domestic economy.

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