COST PADDING IN REGULATED MONOPOLIES

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This paper considers a regulated monopoly that can pad or falsify its costs to increase its cost reimbursement from a regulator. The firm can also engage in a cost reducing investment before it enters into a regulatory contract. The investment in cost reduction determines the firm type and the paper derives the optimum incentive compatible falsification contracts and an equilibrium for the type distribution. It shows that at the optimum price setting regulation is relaxed and the regulator tolerates some cost padding. There is under-investment in cost reduction and investment is distorted away from the cost minimizing level. It also shows that where there is an equilibrium type distribution it is continuous and there are no mass points.

KEYWORDS: Cost padding · costly state falsification · endogenous screening

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1. INTRODUCTION

One of the most serious concerns of regulators is that regulated firms may engage in cost padding or accounting contrivances to increase their costs and hence their remuneration from the regulator.[1] Firms have many ways of padding costs: increasing salaries and expense claims, "gold-plating" of expenditures, charging other equipment to project costs, advertising for corporate image, charging for depreciated assets, not reporting cost reducing improvements and so on. In turn this may reduce the firms' incentive to invest in cost reduction. Despite this most models of regulation are not designed to address the issue of cost padding.[2] The purpose of this paper is

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[1] For examples of such regulatory concerns related to public utilities, the reader is referred to McAfee and McMillan (1988; government contracting - North America), Quiggin (1998; electricity - Australia); Kerr (1998; water - New Zealand); Department of Transportation and Regional Services (2000; transport - Australia); Ontario Federation of Agriculture (1999; energy - Canada); Watson (2000; public utilities - Australia), and the OECD study by Gonenc *et al.* (2000) that compares the incentives that price-cap regulation provides for cost-padding in electricity and telecommunications. For examples in procurement contracts, the reader is referred to Manoj (2000; Shipping - India) and Higgs (1998; military - U.S.A).

[2] There is a small literature, for example Albon and Kirby (1983) and Daughety (1984) which has previously examined cost padding in models with an exogenous regulatory constraint. Waterson (1988) provides a summary of models of this type.

to examine cost padding within the framework of a standard regulatory model and in particular examine how cost padding affects the incentives for investment in cost reduction.

The stated concerns of regulators suggest that the extent of cost padding and the consequent inefficiency is large. It is however, quite difficult to quantify as by definition cost padding is hidden from the regulator and other observers. Despite this the evidence from the classic studies of Berliner (1957), Schiff and Lewin (1968) and Schiff and Lewin (1970) is that there is widespread exaggeration of costs in regulated systems[3] and there are numerous case studies which show how and to what extent regulated firms pad their costs.[4] Since cost padding is largely hidden such cases may represent the tip of an iceberg and therefore it is important to consider the implications of cost padding for efficiency in regulated firms.

Whilst cost padding is an important practical consideration for regulators the main theoretical models of regulation have not designed to address this question.[5] In Baron and Myerson (1982) for example it is assumed that the regulator is unable to observe the firm's costs and therefore since the payment to the firm does not depend upon costs the firm has no incentive to pad them. Equally in the standard model of Laffont and Tirole (1993) it is assumed that the regulator observes the firm's costs perfectly but is unable to observe the firm's effort in cost reduction. Again the firm has no incentive to pad costs as the true cost is perfectly and costlessly monitored.

This paper therefore considers an intermediate case where the regulator can perfectly observe total cost, that is true cost plus padded costs but cannot disentangle the two components. The approach we adopt is to re-interpret the Laffont-Tirole model to allow for a difference between real and observed costs at the contracting

[3] Berliner interviewed former managers of Soviet firms and found as one manager reported "an enormous amount of falsification in all branches of production and in their accounting systems..." (p.161). Schiff and Lewin studied the efficiency of divisions within three large U.S. corporations and produced estimates of the size of cost-padding within divisions of the same company to be between 20 and 25%.

[4] The report by the Godbole committee into the Enron-owned Dabhol Power Company in Maharashtra State, India found cost padding of Rs 930 crore (about $200 million). In the 1989 case of the US government versus the defense contractor Sundstrand, a sum of $200 million was recovered as the court found that Sundstrand had co-mingled commercial and government costs. In a 1985 case a U.S. federal grand jury indicted the General Electric Company on charges that it had falsified claims for work on a nuclear warhead system of at least $800,000. In 1969 the armed services board of contract appeals found that McDonnell-Douglas had overcharged the U.S. Military by over $54,235 by failing to disclose actual experienced manufacturing hours, information about inventories and up to date prices the latest available prices and quotations on purchased parts. The 1984 audit by the U.S. Department of Defense inspector-general found that contractors were inflating charges for spare parts and tools in over one third of all cases, amounting to about a 6% increase in costs.

[5] For a survey of these models see e.g. Laffont (1994).

stage and study the incentives for cost reduction by allowing the firm to undertake a cost-reducing investment at a pre-contractual stage.[6] Padded costs, which are the difference between real and observed costs, are treated as a post-contractual hidden action of the firm chosen to increase its cost reimbursement whereas the pre-contractual investment in cost reduction is treated as the hidden information of the firm at the contracting stage. In treating cost padding as a hidden action of the firm we follow the literature on costly-state falsification initially proposed and analyzed by Lacker and Weinberg (1989).[7] In determining the investment in cost reduction at the pre-contractual stage we follow the approach of González (2004) and Gul (2001) on endogenous screening and derive the distribution of cost types at the contractual stage as an equilibrium outcome of the game played between the firm and the regulator.[8]

When the firm invests in cost reduction at the pre-contractual stage it faces a hold-up problem at the contracting stage. This arises because if the regulatory authority knows or infers the investment level it will be able to design the contract to extract rent from the firm. Anticipating this the firm optimally adopts a mixed strategy and this, as we shall show, lowers average investment. There are two things to note here. First, in contrast to the standard result of Averch and Johnson (1962) where the regulated firm overinvests to meet a regulatory constraint, here investment is reduced relative to the cost minimizing level. Second although this result is similar to that in González (2004) we provide a generalization of his result. Whereas González (2004) restricts attention to continuous strategies we shall allow firms to adopt any mixed strategy and prove that in equilibrium the firm's optimum mixed strategy will have a continuous distribution with no mass points. The intuition why the equilibrium distribution is continuous is that if were not continuous then a firm could profitably fill the gap by choosing an intermediate investment and pad costs thereby increasing its profits. Mass points are also ruled out by showing that a firm can always profitably smooth out the distribution by shifting weight to eliminate the discontinuity.

[6] A similar reinterpretation of the Laffont-Tirole model to allow for cost padding appears in the recent paper of Chu and Sappington (2007). There the focus is an asymmetry in the cost padding and cost reduction technologies at the contracting stage and that paper does not examine the incentives for cost reduction at the pre-contractual stage.

[7] The costly state falsification model has been used and extended in a number of other papers. For example, Maggi and Rodríguez-Clare (1995) consider a general agency model with risk neutral principal and agent, Crocker and Morgan (1998) examine falsification and fraud in insurance contracts under risk aversion and Crocker and Slemrod (2007) which analyzes the relationship between earnings manipulation and executive compensation.

[8] The work of Spiegel and Spulber (1997) considers a different problem of how a firm's capital structure might signal information about its costs to the regulator.

Having determined the equilibrium distribution and the optimum regulatory contract we can examine some comparative statics of cost padding. A natural question to ask is how cost padding and welfare changes if the regulator is able to improve its monitoring technology and thereby raise the costs to the firm of falsifying its costs. Such an increase might, for example, be achieved by a policy that tightens accountancy standards. We show that such changes have both a direct and indirect effect on cost padding. The direct effect reduces cost padding as it has become more expensive for the firm to pad costs. There is however, an indirect effect as the firm has a reduced incentive to engage in pre-contractual cost reduction. We shall indeed show that this indirect effect reduces investment in cost reduction in the sense of first-order stochastic dominance. Although the indirect effect will tend to offset the direct effect, it will be shown that at the optimum the direct effect will dominate and both expected levels of cost padding and falsification costs will be lower.

We shall also demonstrate the importance of accounting for the indirect effect of changes in pre-contractual investment in cost reduction. If say, the distribution of cost types is given exogenously then firms do not respond to increases in the cost of falsification by reducing cost padding (as in this case cost padding depends only on the relative curvature of the cost of falsification function) and expected falsification costs will rise. Hence there may be dramatic differences in the welfare implication of a policy change which improves monitoring technologies or accounting standards between the two cases where the incentive effects on pre-contractual investment are taken into account and where they are not. We shall give an example to illustrate that the welfare conclusions in these two cases can be diametrically opposed.

The paper proceeds as follows. Section 2 outlines the model and Section 3 provides the results. Section 3.3 solves for the optimum contract with a fixed distribution of types and Section 3.4 solves for the equilibrium type distribution determined by the pre-contractual investment. Section 3.5 studies some comparative static properties of the solution and Section 3.6 considers the welfare implications of a policy which raises falsification costs in the two cases where the type distribution is fixed and when it is determined endogenously. Section 4 concludes. Proofs omitted from the text can be found in the Appendix.

2. The Model

In this section we describe the regulatory model. The regulator wishes to engage a firm to undertake a project to maximize social welfare. Before the regulator engages the firm, the firm chooses the amount of capital to invest which affects the cost of undertaking the project. Once the firm is engaged it is able to pad its costs. The regulator observes the firm's total cost for the project but does not observe how much capital the firm has invested nor how total cost is divided between real and padded costs. The regulator has to design a contract for the firm which discourages it from padding costs and encourages it to invest in cost reduction. Thus we change the order of the standard Laffont and Tirole (1993) regulatory model and assume that the effort or investment in cost reduction is pre-contractual and that the firm can artificially inflate or pad its cost at the post-contractual stage after production is undertaken. At the contractual stage the regulator treats the firm's investment in cost reduction as *hidden information* and the padded costs of the firm as a *hidden action*.

The regulator chooses the size of the project q for the firm is to deliver. A project of size q has social benefit $V(q)$ where $V: \mathscr{Q} \to \Re$ and $\mathscr{Q} = [0, q_{max}] \subset \Re_+$.[9] There is a shadow cost of public funds of $1+\lambda$. We make the standard assumptions on social benefit and the cost of public funds.

ASSUMPTION 1: $V: \mathscr{Q} \to \Re$ is C^3, $V(0) = 0$, $V'(q) > 0$, $V''(q) < 0$, satisfies the Inada condition $\lim_{q \to 0} V'(q) = \infty$ and $\lambda > 0$.

If the project is to produce a private good we shall denote the inverse demand function as $P(q)$ where $P: \mathscr{Q} \to \Re_+$. Letting $R(q) = qP(q)$ be firm revenue and $S(q)$ be *gross* consumer surplus, $V(q) = (S(q) - R(q)) + (1+\lambda)R(q) = S(q) + \lambda R(q)$. As $S'(q) = P(q)$, the marginal social benefit is $V'(q) = (1+\lambda)P(q) + \lambda q P'(q)$.

The firm chooses its capital from an interval $\mathscr{K} = [0, k_{max}] \subset \Re_+$ before the project contract is signed. The market for capital is perfect and the price of capital is $\rho > 0$. The cost to the firm of undertaking the project depends on the size of the project q and the amount of capital k devoted to the project. Denote the firm's variable cost as $g(k,q)$ where $g: \mathscr{K} \times \mathscr{Q} \to \Re_+$. The firm's short-run marginal cost is therefore $g_q(k,q)$. We make the following assumptions about the firm's variable cost function:

ASSUMPTION 2: $g: \mathscr{K} \times \mathscr{Q} \to \Re_+$ is C^3 and convex, $g_q(k,q) > 0$, $g_k(k,q) < 0$, $g_{qq}(k,q) > 0$, $g_{kq}(k,q) < 0$, $g_{kk}(k,q) > 0$ and $-g_k(0,q) > \rho$ for $q > 0$.

REMARK: Here we have chosen to take the variable cost function as a primitive of the model. With two minor qualifications we could have taken the firm's production function, say with two inputs, capital k and labor ℓ, as a primitive and derived the properties of the variable cost function given in the assumption. The qualifications concern the sign of $g_{kq}(k,q)$ and the continuity of the derivatives of the function at the boundary. Firstly, $g_{kq}(k,q)$ has the same sign as $(f_{\ell\ell} f_k - f_{\ell k} f_\ell)$. In general the cross-derivative $g_{kq}(k,q)$ may be of either sign, However, for many production functions, such as the Cobb-Douglas and CES cases, it is negative. Secondly if Inada

[9] We shall also later be interested in the procurement case where $\mathscr{Q} = \{0, 1\}$.

conditions are imposed on the production function then the derivatives of the cost function, in particular $g_k(k,q)$, may be unbounded and hence not continuous at $k=0$. The later raises a technical issue which we avoid by making assumptions directly on the cost function.[10] The last part of the assumption is a non-triviality condition to ensure that investment is strictly positive at the first-best solution.

We shall also be interested in the special case where g is *multiplicatively separable* in k and q.

ASSUMPTION 2′ : $g(k,q) = g^0(k)g^1(q)$ for some functions $g^0\colon \mathscr{K} \to \Re_+$ and $g^1\colon \mathscr{Q} \to \Re_+$.

This separability assumption is satisfied if the cost function is derived from the Cobb-Douglas production function. Under Assumption 2′ the elasticity of costs with respect to k, i.e. $k\,g_k(k,q)/g(k,q)$, is independent of q.

The regulator cannot observe the true variable cost but instead observes total costs C which also includes the level of cost padding $x \geq 0$ undertaken by the firm, i.e. $C = g(k,q)+x$. Thus the amount x represents the extent to which the firm falsely reports its costs[11] and can be considered as an accounting contrivance that raises the costs as seen by the regulator. We shall also be interested in the proportion of costs padded and let χ satisfy $C = (1+\chi)g(k,q)$ so that χ represents the extent to which input prices are exaggerated by the firm. This is consistent with a number of the examples cited in the introduction where the prices of inputs as billed to the regulator were marked up relative to true cost. We follow the standard costly-state falsification model of Crocker and Morgan (1998) and assume that there is a cost to falsifying the reported costs of $\phi(x)$ that depends on the extent of cost padding x. The following assumption is made about the function $\phi(x)$:

ASSUMPTION 3: $\phi\colon \Re_+ \to \Re_+$ is C^3, $\phi(0) = 0$, $0 \leq \phi'(x) < 1$, $\phi'(0) = 0$, $\phi''(x) > 0$.

These assumptions are mainly straightforward. The marginal cost of falsification is positive and increasing so that more exaggerated costs are increasingly costly to

[10] What is required is that functions are integrable. This will not in general be true for standard definitions of integrability if the functions are not continuous at the boundary. Nevertheless in the examples we present below the cost functions are derived from a Cobb-Douglas production function and even though the resultant cost functions have unbounded derivatives, the solutions can be correctly computed.

[11] There is another interpretation for x as additional or unnecessary expenditures undertaken by the firm and this will be discussed below. As noted in the introduction there are many ways in which firms can pad costs: advertising and sponsorship, transfer of funds across divisions, unnecessary remuneration increases, larger than normal allowances for depreciation, not reporting on cost-saving improvements, and various other perks as well as other costly accounting contrivances.

falsify. It is assumed that $\phi'(x) < 1$ so that the marginal cost of falsification is always less than the amount falsified. This is assumed here but could easily be imposed as a constraint as it will never be profitable for the firm to falsify the extra unit of cost if the cost of the falsification is greater than any potential benefit. A convenient functional form that satisfies Assumption 3 arises when the elasticity of marginal cost is constant.

ASSUMPTION 3$'$: $\phi(x) = (\alpha/\beta)x^{\beta}$ for $\alpha > 0$, $\beta > 1$ and $x \in [0, \alpha^{1/(1-\beta)}]$.

We now consider the sequential move game in which the firm first chooses its capital input and then the regulator chooses an incentive contract for the firm given the hidden action problem that the firm can pad its costs. We suppose that the firm can chose a mixed strategy and represent its mixed strategy choice by a distribution function $F : \mathscr{K} \to [0,1]$.[12] The distribution function is by definition non-decreasing and right-continuous. The distribution may however, have positive density, zero density or positive probability. By the Lebesgue decomposition theorem we know that every distribution function can be written as a convex combination, $F = \alpha_c F_c + \alpha_d F_d + \alpha_s F_s$, where F_c is absolutely continuous, F_d is discrete, F_s is singular continuous (i.e. a distribution which is continuous but non-increasing almost everywhere such as the Cantor distribution) and where $\alpha_d, \alpha_c, \alpha_s \geq 0$ and $\alpha_d + \alpha_c + \alpha_s = 1$. We shall let $F(k-) \equiv \lim_{h \searrow 0} F(k-h)$ be the left hand limit where the notation $\lim_{h \searrow 0}$ indicates that the limit is approached as h tends to zero for $h > 0$. Since the distribution is right continuous $F(k+) \equiv \lim_{h \searrow 0} F(k+h) = F(k)$. Also since the distribution function is monotonic it can have at most a countable set of discontinuities. Let the set of these discontinuities be $\mathscr{M} = \{k_0, k_1, k_2, \ldots\}$. Then we shall let $f(k_i) \equiv F(k_i) - F(k_i-) > 0$ denote the mass or saltus at k_i. If the firm adopts a pure strategy k_0 say, then the distribution has a single mass point with $f(k_0) = 1$. We shall denote the support of F, i.e. the set of all the points where F is strictly increasing, as $\mathscr{S} \subseteq \mathscr{K}$.[13]

Denote $\mathscr{C}$ as the set of possible costs. Given that the regulator can observe the total costs it will offer an incentive contract conditional on $C \in \mathscr{C}$. As a convention suppose that the regulator reimburses the firm's total cost C and that the contract specifies a transfer $t \in \Re$ to the firm as well as the size of the project $q \in \mathscr{Q}$. Note that since reimbursed cost includes padded costs, the transfer t may be (and in some cases will be) negative. The rent r that the firm gets from the contract is given by the

[12] A mixed strategy is a probability function on $\mathscr{K} \subset \Re$ and this is uniquely related to its distribution function.

[13] The support of a distribution is the minimal closed set whose complement has probability zero. Each element of the support is a point of increase. That is a point k such that $F(k+h) - F(k-h) > 0$ for all $h > 0$. Otherwise the probability $\Pr(k-h, k+h) = 0$ in which case k does not belong to the support.

transfer t plus the reimbursed costs observed by the regulator C, less the true variable costs of production $g(k,q)$, less the cost of falsifying the accounts $\phi(C-g(k,q))$. That is[14]

$$r = t + C - g(k,q) - \phi(C - g(k,q)).$$

The regulator's objective is to maximize total welfare, including the rents of the firm, net of costs. This is given by

$$V(q) - (1+\lambda)(g(k,q) + \phi(C - g(k,q))) - \lambda r - \rho k$$

where he first term is the social benefit of the project, the second term is total social cost, the third term is the social cost of giving the firm a rent and the final term is the capital cost.[15]

Denote the costs which the firm incurs as $\upsilon(C,k,q) = g(k,q) + \phi(C - g(k,q))$ and let $\omega(C,k,q) = V(q) - (1+\lambda)\upsilon(C,k,q)$ denote the net social benefit of the project. Given Assumption 2 and Assumption 3, the function υ is strictly decreasing and strictly convex in k and satisfies a Spence-Mirrlees *single crossing property*. For convenience we state this as a separate assumption.

ASSUMPTION 4 (Single Crossing Property): The marginal cost υ_k satisfies the single crossing property: $\upsilon_{kC} > 0$ and $\upsilon_{kq} < 0$.

Further to ensure that the regulator's optimization problem is concave and has a global interior solution we shall make an additional assumption.

ASSUMPTION 5: Both the benefit function $\omega(C,k,q)$ and the marginal cost function $\upsilon_k(C,k,q)$ are assumed to be strictly concave in C and q.

Assumption 5 clearly implies further restrictions on the benefit and cost functions beyond those of Assumption 1, 2 and 3 and imposes restrictions on the third derivatives of the cost function g and cost falsification function ϕ (all of which are satisfied by the examples we compute below). In particular Assumption 5 implies that $\phi''' \leq 0$ so that the marginal falsification costs are also concave. Moreover Assumption 5 is sufficient to assure that random contracts are sub-optimal.

As we have stated, the regulator observes C but not the padded cost x Thus no inferences can be drawn by the regulator about the level of capital k chosen by the

[14] A participation constraint will be imposed so that the rent is non-negative. Hence even when the transfer t is negative, the total transfer $t+C$ is sufficient to cover costs.

[15] The cost of capital is included in the total welfare for later comparisons even though k is fixed at the time of contracting and therefore won't affect the regulator's optimization problem.

firm. We therefore assume that the regulator forms a probability assessment about the choice of capital input made by the firm and represent this by a probability distribution $F^e : \mathscr{K} \to [0,1]$ with support $\mathscr{S}^e$. Given this probability assessment the regulator will design the transfer and project size to maximize expected social welfare by making the transfer t and size q depend upon the observed costs C so as to reduce the inefficiencies caused by the firm's potential to cost pad. This is a very standard hidden action problem and it is possible to apply the revelation principle to restrict attention to direct mechanisms where the firm reports its investment in cost reduction k' and impose incentive compatibility constraints that the firm has no incentive to misreport its investment level. We shall say that the regulator's probability assessment is *consistent* if $F^e = F$.[16] Therefore the regulator's problem can be formulated by allowing the regulator to choose a contract $\Delta \equiv (C(k), q(k), r(k))_{k\in\mathscr{K}}$ which specifies the cost $C(k)$, the project size $q(k)$ and the rent $r(k)$ as functions of investment k to maximize expected social welfare

$$(1) \qquad \int_{\mathscr{K}} \{V(q) - (1+\lambda)\upsilon(C,k,q) - \lambda r(k) - \rho k\}\, \mathrm{d}F^e(k)$$

subject to the incentive compatibility and participation constraints for the firm. Define the function $r(k',k) = t(k') + C(k') - \upsilon(C(k'),k,q(k'))$ be the rent which the firm earns from the contract when the actual investment is k and when the firm reports that it has invested k' for each $k' \in \mathscr{S}^e$[17] and let $r(k) = r(k,k)$. The firm will choose to announce the level of cost reducing investment k' that maximizes the rent given the contract Δ it faces. Therefore the incentive compatibility constraints are

$$(2) \qquad r(k) \geq r(k',k) = t(k') + C(k') - \upsilon(C(k'),k,q(k')) \qquad \forall\ k,k' \in \mathscr{S}^e.$$

Given incentive compatibility and since the investment in cost reduction is a sunk cost, for the firm to participate in the contract it must always derive a non-negative rent for any investment level k actually made. That is

$$(3) \qquad r(k) \geq 0 \qquad \forall\ k \in \mathscr{S}^e.$$

An equilibrium will involve a regulatory contract which maximizes social welfare subject to the incentive compatibility and participation constraints, a strategy for the

[16] For an analysis of screening where the beliefs of the principal and agent are not consistent see Grubb (2009). One rationale for focusing on consistent beliefs is that in the long-run, and facing many similar situations, the regulator will correctly anticipate the firm's mixed strategy.

[17] We assume that if $k' \notin \mathscr{S}^e$ then the regulator will make no transfer to the firm. Hence such announcements will not be optimal for the firm. The firm may however, choose some $k \notin \mathscr{S}^e$.

firm which maximizes profits (rent less capital costs) and a consistent probability assessment. If the firm is to adopt a mixed strategy in equilibrium, each possible choice of investment level k must also give rise to the same level of profit. That implies that in any equilibrium

$$r(k) - \rho k = r(k') - \rho k' \qquad \forall k, k' \in \mathscr{S}. \tag{4}$$

We shall further assume that if the firm is indifferent between two or more strategies then it will choose the strategy that maximizes the regulator's objective.[18] Hence we can define an equilibrium as follows:

DEFINITION 1: A equilibrium will be a contract Δ which maximizes (1) subject to (2) and (3), a strategy for the firm that maximizes profits and a consistent probability assessment $F^e = F$. Where the firm is indifferent between two or more strategies it chooses the strategy which maximizes the regulator's payoff. In addition if the firm chooses a mixed strategy equation (4) is also satisfied.

Before proceeding to the analysis it is important to realize that although in describing the model we have treated cost padding as an accounting contrivance, there is an equivalent model in which cost padding can be interpreted as an extra but unnecessary expenditure that generates some utility benefit for the firm. To see this let $\psi(x)$ denote the benefit to the firm of cost padding an amount x. In this case ψ may be thought of as the benefit of gold-plating expenditures. Assume that the benefit function $\psi(x)$ satisfies $\psi(0) = 0$, $0 \leq \psi'(x) \leq 1$, $\psi'(0) = 1$ and $\psi''(x) < 0$. That is an increase in padded costs by one unit generates a positive gain in utility (but not as much as the costs incurred) and marginal benefit is declining with costs. The rent of the firm r, is simply given by the transfer t plus the utility benefit of the padded costs $\psi(x)$ so that rent is given by $r = t + \psi(x)$. Then the two alternative interpretations are formally equivalent if $\psi(x) = x - \phi(x)$.[19] Although the two interpretations are formally equivalent in this case, it should be emphasized that these two alternatives represent very different situations. In one case there is a real expenditure that generates utility benefits whereas in the other case it is an accounting contrivance which has real costs.

3. RESULTS

The results are organized as follows. Section 3.1 considers the benchmark first-best solution where the regulator can observe the true cost. Section 3.2 develops

[18] If preferred this may be viewed as imposing an upper bound on the regulator's payoff.

[19] In the previous case rent can be rewritten as $r = t + x - \phi(x)$ and the assumptions on $\psi(x)$ imply the conditions given in Assumption 3 on the function $\phi(x)$.

some preliminary results so that the optimum contract can be examined. Section 3.3 assumes that the firm adopts some mixed strategy and derives the optimum contract given that mixed strategy. Section 3.4 extends the analysis by deriving the equilibrium distribution. Section 3.5 considers some comparative statics and Section 3.6 compares the solution with the endogenous distribution of types with that with an exogenous distribution.

3.1. The First-Best

As a benchmark consider the case where the regulator can observe the true cost (and hence can infer k) so there is no hidden action or hidden information problem. In this case there is no cost padding and the first-best optimum is to set the marginal social value of an extra unit of output equal to the short run marginal cost of public funds. The short run marginal cost of public funds is $(1+\lambda)g_q(k,q)$ and the condition for the first-best optimum is

$$V'(q) = (1+\lambda)g_q(k,q). \tag{5}$$

In addition the firm will choose the capital input to equate the marginal benefit of cost reduction for a given project level to its price

$$-g_k(k,q) = \rho. \tag{6}$$

We shall let k^* and q^* denote the first-best levels of capital and project size that satisfy (5) and (6). It will also be useful to denote the first-best level of output for a given capital input which solves (5) as $q^{FB}(k)$. Likewise denote the first-best level of investment for any given output level which satisfies (6) as $k^{FB}(q)$. It is straightforward to see from Assumption 1 and Assumption 2 that both $q^{FB}(k)$ and $k^{FB}(q)$ are strictly increasing. Also by definition $q^* = q^{FB}(k^*)$ and $k^* = k^{FB}(q^*)$.

LEMMA 1: *Given Assumption* 1 *and* 2*, there is a unique pair* (k^*, q^*) *with* $q^* > 0$ *and* $k^* > 0$ *which solve equations* (5) *and* (6)*.*

PROOF: Solving for $k^{FB}(q)$ from (6) and substituting into (5), it follows from the strict concavity of V and the convexity of g that the $V'(q) - (1+\lambda)g_q(k^{FB}(q),q)$ is strictly decreasing in q. Since $\lim_{q\to 0} V'(q) = \infty$, $q^* > 0$. It follows from Assumption 3 that $-g_k(0, q^{FB}(0)) > \rho$ and so $k^* > 0$. □

For a private good, let the Lerner index be given by $L = (p - g_q(k,q))/p$ and the Ramsey index be given by $R = (\lambda/(1+\lambda))\eta^{-1}$ where $\eta = -(dq/dp)/(q/p)$ is the elasticity of demand. At the social optimum (k^*, q^*), the Ramsey pricing rule $L = R$

holds and the Lerner index is proportional to the inverse of the elasticity of demand with the factor of proportionality $\lambda/(1+\lambda)$.

3.2. Preliminaries

We first rule out the existence of a pure strategy. Suppose the regulator were to believe that the firm chooses some $\hat{k} > 0$ with probability one. The optimal regulatory contract in this case is for the regulator to choose the first-best project size of $q^{FB}(\hat{k})$, reimburse the cost $C(\hat{k}) = g(\hat{k}, q^{FB}(\hat{k}))$ and make no transfer to the firm. This however, cannot be an equilibrium as if the firm chose $\hat{k}$ it would pad no costs and receive no rent from the contract leading to profits of $-\rho\hat{k} < 0$. Thus a deviation to choosing zero investment would increase profits.[20] Thus the only candidate for a pure strategy equilibrium is where the regulator believes the firm chooses $k = 0$ with probability one. In this case the regulator chooses a contract with output $q^{FB}(0)$ and reimburses costs $C(0) = g(0, q^{FB}(0))$. Consider then the deviation where the firm marginally increases investment by $dk > 0$ and pads costs so that total costs remain at $C(0)$. Real costs decrease by $g_k(0, q^{FB}(0))dk$ and investment costs increase by ρdk. The cost of padding costs is zero on the margin (from Assumption 3 that $\phi'(0) = 0$). Hence the increase in profits from the deviation is $(-g_k(0, q^{FB}(0)) - \rho)dk$, which is positive by Assumption 3 and so the deviation is profitable.

Having established that there is no pure strategy equilibrium, we shall from now on consider situations in which the firm plays a mixed strategy F. For such a mixed strategy it is easy to establish the standard result that in any optimal contract the local downward incentive compatibility constraints of equation (2) are binding.[21]

LEMMA 2: *Given a distribution for the regulator's beliefs F^e then for any two isolated but adjacent values in the support, the downward incentive compatibility constraints bind at the optimum contract.*

Using this property it then follows that if the firm adopts a mixed strategy, the support of the regulator's beliefs must be an interval in equilibrium and hence a connected set. Thus we can conclude that $\alpha_s < 1$ so the distribution cannot just be composed of a singular continuous element (which has a nowhere dense support). The idea is simple. If the support is not an interval then the firm can choose an intermediate investment level not in the support, report a lower investment level within the support and increase profits.

LEMMA 3: *If the regulator's beliefs are consistent then the set $\mathscr{S}^e$ is a closed interval.*

[20] There may be other profitable deviations too.

[21] As stated above proofs omitted from the text can be found in the Appendix.

Having shown that the support $\mathscr{S}^e$ is a closed interval denote its lower endpoint by $\underline{k}$ and its upper endpoint by $\bar{k}$. We shall determine the values of $\underline{k}$ and $\bar{k}$ in Section 3.4. Since the continuous part of the distribution F_c is monotonic on this interval it is differentiable almost everywhere. As normal we can define a density function f_c for the continuous part of the distribution function to equal the derivative F_c' where the derivative exists and zero otherwise. Then given the support is an interval, we can write the regulator's objective function as

$$(1') \qquad \int_{\underline{k}}^{\bar{k}} (V(q(k)) - \lambda r(k) - (1+\lambda)\upsilon(C(k),k,q(k)) - \rho k)\, \mathrm{d}F^e(k)$$

where the differential is

$$dF^e(k) = \begin{cases} F^e(k_i) - F^e(k_i-) & \text{if } k_i \in \mathscr{M}^e \\ f_c^e(k) & \text{if } k \in \mathscr{S}^e/\mathscr{M}^e \end{cases}$$

Further since the support is an interval and hence connected we can use the fairly standard procedure (see e.g. Segal and Whinston (2002)) to give necessary and sufficient conditions for incentive compatibility.

LEMMA 4: *Necessary and sufficient conditions for incentive compatibility are*

(i) $\qquad r(k) = r(\underline{k}) - \int_{\underline{k}}^{k} \upsilon_k(C(\kappa),\kappa,q(\kappa))\,\mathrm{d}\kappa$

(ii) $\qquad -\int_{k}^{k'} (\upsilon_k(C(k'),\kappa,q(k')) - \upsilon_k(C(\kappa),\kappa,q(\kappa)))\,\mathrm{d}\kappa \geq 0 \quad \forall k$ *and* k'.

The first part of Lemma 4 shows that higher rent must be paid to a low cost firm (high k) to induce them to report their lower costs since $\upsilon_k(C,k,q) < 0$. This higher rent will exactly reflect the reduction in costs. As k is increased by one unit true cost falls by $g_k(k,q)$, but for a given C there is a partially offsetting increase in falsification costs of $\phi'(C-g(k,q))g_k(k,q)$ and a corresponding increase in rent of $\upsilon_k(C,k,q(k)) = g_k(k,q)(1-\phi'(C-g(k,q)))$. Moreover Lemma 4(i) shows that the derivative $\dot{r}(k)$ satisfies

$$(2') \qquad \dot{r}(k) = -\upsilon_k(C(k),k,q(k))$$

wherever $r(k)$ is differentiable. The second part of Lemma 4 is the usual second-order condition expressed in integral form. We shall follow the standard procedure of ignoring this condition and then checking it is satisfied by the optimum contract. Thus equation (2′) replaces all the incentive compatibility conditions of equation (2).

Likewise it is possible to follow a standard argument to simplify the participation constraint given in equation (3). Since the rent enters negatively into the objective function (1) with $\lambda > 0$ and since $\dot{r}(k) \geq 0$ from (2′), the regulator gains by a parallel downward shift in the rent function $r(k)$ so that the participation constraints can be replaced by the single equality condition

$$r(\underline{k}) = 0. \tag{3′}$$

Taking these simplifications into account the problem for the regulator is to choose a contract $\Delta = (C(k), q(k), r(k))$ to maximize (1′) subject to (2′) and (3′). Integrating $r(k)dF^e(k)$ by parts, using Lemma 4(i), (2′) and (3′) and substituting into (1′) gives the virtual surplus function

$$\int_{\underline{k}}^{\bar{k}} (V(q) - (1+\lambda)\upsilon(C,k,q) - \rho k)\, \mathrm{d}F^e(k) + \int_{\underline{k}}^{\bar{k}} \lambda\, \upsilon_k(C,k,q)(1 - F^e(k))\, \mathrm{d}k. \tag{1″}$$

3.3. Optimal Contract with Fixed Distribution

Since we shall only consider consistent distributions we henceforth drop the superscript notation e to distinguish the regulator's beliefs from the actual distribution of k. The following first-order conditions are derived straightforwardly from equation (1″)

$$(1+\lambda)\upsilon_C(C,k,q)dF(k) = \lambda\, \upsilon_{kC}(C,k,q)(1-F(k)) \tag{7}$$

$$\big(V'(q) - (1+\lambda)\upsilon_q(C,k,q)\big)\, dF(k) = -\lambda\, \upsilon_{kq}(C,k,q)(1-F(k)). \tag{8}$$

These two equations can be written more illuminatingly as

$$(1+\lambda)\phi'(x(k))dF(k) = -\lambda(1-F(k))g_k(k,q)\phi''(x(k)) \tag{7′}$$

$$\big(V'(q) - (1+\lambda)g_q(k,q)\big)\, dF(k) = -\lambda(1-F(k))(1-\phi'(x(k)))g_{kq}(k,q) \tag{8′}$$

where equation (8′) has been simplified by using both equations (7) and (8) and $C - g(k,q)$ is replaced by x. At the first-best solution the left-hand-sides of both equations (7′) and (8′) are equal to zero. Thus the equations show how the optimal contract differs from the first-best solution. Consider first equation (7′). The benefit of reducing cost padding by one unit is simply the saving in falsification costs $\phi'(x)$. If cost padding is reduced for $dF(k)$ cost types, the benefit to society is $(1+\lambda)\phi'(x)dF(k)dk$ as the shadow cost of the funds saved is $(1+\lambda)$. The cost of this one unit reduction is however, the extra rent determined by equation (2) that must be paid to all types in the interval $[k,\bar{k}]$ to induce them to report lower costs. The change in the rent for a firm with capital k following a one unit reduction in padded costs is $\phi''(x)g_k(k,q(k))dk$. The social cost of an extra rent payment of one unit is λ and since $(1-F(k))$ cost types are affected the social cost of the extra rent payments is $\lambda(1-F(k))\phi''(x)g_k(k,q(k))$. Equating the marginal social cost to the marginal social benefit gives equation (7′).

The interpretation of equation (8′) is equally straightforward. A small change in q changes the social benefit by $V'(q)$ and changes social cost by $(1+\lambda)g_q(k,q)$ for any given level of capital, k. The extra term in equation (8′) represents the marginal increase in rent needed by the more efficient cost types to maintain incentive compatibility. The effect on the rent of more efficient firms is $-(1-\phi'(x))g_{kq}(k,q)$ which is positive for a small increase in output given Assumption 2 that more efficient types have lower marginal costs as well as lower average costs, i.e. $g_{kq}(k,q) < 0$. This cost of extra rent has to be weighted by the social cost λ and the proportion of more efficient types there are, $1-F(k)$, relative to the proportion of cost types, $dF(k)$, with capital k. Thus equating the marginal social benefit to the marginal social cost plus the marginal social incentive cost gives equation (8′). We can therefore conclude from these two equations the following proposition.

PROPOSITION 1: *Given Assumptions* 1, 2 *and* 3*, if there is an equilibrium in which the firm adopts a mixed strategy $F(k)$, costs are padded, $x(k) = C(k) - g(k,q(k)) > 0$, and output is restricted, $q(k) < q^{FB}(k)$, for each $k < \bar{k}$.*

PROOF: $dF(k) > 0$ and $F(k) < 1$ for $k < \bar{k}$. Since $\phi''(x) > 0$ and $g_k(k,q(k)) < 0$, then $\phi'(x) > 0$ and costs are padded. To see that output is restricted recall the assumptions $\phi'(C(k)-g(k,q(k))) < 1$ and $g_{qk}(k,q(k)) < 0$, so that the marginal social incentive cost is positive. Thus

$$V'(q(k)) - (1+\lambda)g_q(k,q(k)) > V'(q^{FB}(k)) - (1+\lambda)g_q(k,q^{FB}(k))$$

for $k < \bar{k}$. Then since $V''(q) < 0$ and $g_{qq}(k,q) > 0$, we have that output is restricted below the efficient level. □

It is important to remember that although Proposition 1 shows that costs are padded and output restricted below the first-best this does not necessarily mean that the firm isn't cost minimizing. There need be no distortion of input choices (the next subsection will show how input choices may be distorted). It should also be noticed that the regulator will take into account that costs will be padded when setting the contract. Thus a firm that reports a high cost (low k) may be required to make a transfer to the regulator to offset the fact that the regulator reimburses full costs. On the other hand a firm that reports a low cost (high k) may well receive a positive transfer from the regulator.

Equation (7′) also provides information on when there will be no cost padding. Clearly there is no cost padding if $g_k(k,q) = 0$ or $\phi''(x) = 0$. If $g_k(k,q) = 0$ then the unobserved investment does not influence costs. Hence the regulator will know the true cost and there is no room for cost padding. However, there will also be no cost padding if the costs of falsification are linear. This result has been demonstrated by Lacker and Weinberg (1989) who assume a cost function of the type $\phi(x) = \alpha|x|$. The intuition is that if falsification costs are linear then the cost of deterring cost padding is the same independently of the amount of cost padding and therefore no extra rent has to be offered to lower cost types. Thus if any cost padding is to be deterred this will also deter very small amounts of cost padding as well. Also since $\bar{k}$ is in the support, $dF(\bar{k}) > 0$ so there is also the classical efficiency at the top result that there is no cost padding for $k = \bar{k}$. Hence output is also at the first-best level, $q(\bar{k}) = q^{FB}(\bar{k})$ for $k = \bar{k}$. The intuition is that there is always some benefit to reducing falsification at the very highest capital level but the social cost is infinitesimal as no extra rent has to be paid to any other firm type.

For a private good the restriction in output can be expressed in terms of the Lerner and Ramsey indices by rewriting equation (8′) as

$$L = R - \frac{\lambda}{(1+\lambda)}\frac{1}{\eta}\left(\frac{(1-\phi'(C(k)-g(k,q(k)))g_{qk}(k,q(k))(1-F(k))}{pdF(k)}\right).$$

Since $g_{qk}(q,q(k)) < 0$ it can be seen from this equation that the effect of cost padding is to raise the Lerner index above the Ramsey index. Thus cost padding under optimal regulation will tend to restrict output and raise the price. The intuition is fairly simple, since the regulator faces the problem of the firm falsifying its cost upward, it can keep total costs lower by restricting output below the normal Lerner-Ramsey rule, that is by relaxing the price regulation on the firm. As mentioned in the introduction this is a similar conclusion to that found in Daughety (1984) with an arbitrary regulatory constraint.

Under Assumptions 2′ and 3′ equations (7′) and (8′) can be treated independently and hence we have the result that the size of the project does not effect the extent to which input costs are exaggerated.

PROPOSITION 2: *Under Assumptions* 2′ *and* 3′*, if there is an equilibrium in which the firm adopts a mixed strategy $F(k)$, then the proportion of costs padded χ is independent of q.*

PROOF: Using Assumptions 2′ and 3′ in (7′), and recalling the definition $C = (1+\chi)g(k,q)$ it can be seen that χ is independent of q. □

We conclude this sub-section with a two short examples to illustrate the nature of the optimum contract. The first for a purely private good and the second for a procurement case when quantity is fixed.

EXAMPLE 1: Assume that short-run variable costs are $g(k,q) = q^2/4k$, falsification costs are quadratic $\phi(x) = (1/2)x^2$ and that the distribution function is uniform on $[0,1]$. Further assume that demand is unit elastic, $P(q) = 1/q$ and the shadow cost of public funds is $\lambda = 1$ and the cost of capital is $\rho = 1/4$. With these parameters the marginal social benefit of the project is $1/q$ and the first-best solutions are $k^* = q^* = 1$. Equation (7′) shows that $\chi(k) = (1/2)(1-k)/k$. Hence low cost firms will engage in proportionally less cost padding. Equating marginal social benefit with the marginal social cost plus the marginal social incentive cost gives output $q(k) = 2k/(1+\sqrt{k})$[22] and costs $C(k) = (1/2)(1+k)/(1+\sqrt{k})^2$. The expected cost padding is $\mathrm{E}[x] = (3/2) - 2\log_e 2$ and the expected falsification costs are $\mathrm{E}[\phi(x)] = -(11/8) + 2\log_e 2$. Expected total cost is $\mathrm{E}[C] = -(5/2) + 4\log_e 2$ and expected output is $\mathrm{E}[q] = (10/3) - 4\log_e 2$. The expected rent paid to the firm is $\mathrm{E}[r] = (17/2) - 12\log_e 2$ and the expected transfer is $\mathrm{E}[t] = (45/8) - 8\log_e 2$.

EXAMPLE 2: Assume $q \in \{0,1\}$ and that for $q = 1$ the cost function satisfies $g(k) = 2(2-\sqrt{k})$. Assume that falsification costs are quadratic $\phi(x) = (\alpha/2)x^2$ for $\alpha < 1$ and that the distribution function satisfies $F(k) = \sqrt{k}$ on $[0,1]$. We suppose that $\rho = \lambda = 1$. Then $x(k) = 1 - \sqrt{k}$, and rent is $r(k) = 2(1-\alpha)\sqrt{k} + \alpha k$.[23] The total cost is $C(k) = 5 - 3\sqrt{k}$, which since it is decreasing in k means that the second-order condition is satisfied. The transfer function is $t(k) = 3(1-\alpha)\sqrt{k} + (\alpha/2)(1+3k) - 1$. The expected amount of cost padding is $\mathrm{E}[x] = 1/2$ and the expected falsification cost is $\mathrm{E}[\phi(x)] = \alpha/6$. The expected total cost is $\mathrm{E}[C] = 7/2$. The expected rent paid to the firm is $\mathrm{E}[r] = (1-\alpha) + (\alpha/3)$ and the expected transfer is $\mathrm{E}[t] = (1-\alpha)/2$. Investment is undertaken, so $q = 1$, provided $V > 7 - (\alpha/3)$.

3.4. Equilibrium Distribution

As discussed in Section 2 if the firm is to adopt a mixed strategy the profit of the firm must be the same for each level of investment in the support. That is the rent of the firm satisfies $r(k) = const. + \rho k$ for every k in the support. Since by Lemma 3 the support is an interval in any consistent equilibrium and as the profits of the firm must be the same for each possible level of cost reduction we can use the incentive

[22] It can be shown from the solutions for $\chi(k)$ and $q(k)$ that the second-order condition given in Lemma 4(ii) are satisfied.

[23] The assumption $\alpha < 1$ guarantees that $r(k)$ is increasing for each $k \in [0,1]$

compatibility condition of equation (2) to derive

$$(9)\qquad -\upsilon_k(C(k),k,q(k)) = -(1-\phi'(C(k)-g(k,q(k))))g_k(k,q(k)) = \rho.$$

This equation has some straightforward but important implications.

PROPOSITION 3: *If there is an equilibrium where the firm adopts a mixed strategy, then investment is below the first best, $k < k^{FB}(q)$ for all $k < \bar{k}$.*

PROOF: For a given level of output q, (9) shows that $k < k^{FB}(q)$ for all $k < \bar{k}$, since $\phi'(C(k)-g(k,q(k))) < 1$ for $k < \bar{k}$ from (7′) and $g_{kk}(k,q) > 0$. □

REMARK: A corollary of Proposition 3 is that if costs are derived from cost minimization subject to a production constraint, then the marginal rate of technical substitution of capital for labor exceeds its relative price. To see this suppose that cost is derived from a production function, say with other input of labor ℓ with a unit cost. Then $-g_k(k,q) = f_k/f_\ell$ and since equation (9) shows that $-g_k(k,q) > \rho$ costs are higher than the first-best level for any given output and investment. Thus investment is lower than the cost minimizing level for all output levels except $q(\bar{k})$. The implication is that capital accumulation is reduced relative to its cost minimizing level. This is the opposite to the conclusion of the standard Averch and Johnson (1962) model where the marginal rate of technical substitution of capital for labor is less than the relative price of capital, $f_k/f_\ell < \rho$ as the firm over accumulates capital to meet a rate of return constraint on capital.

Next we consider the determination of the endpoints of the support of the distribution, $\underline{k}$ and $\bar{k}$.

LEMMA 5: *In any equilibrium in which the firm adopts a mixed strategy $\underline{k} = 0$ and $\bar{k} = k^*$.*

PROOF: At the pre-contractual stage the firm has the option not to undertake any extra investment in cost reducing activity. With k known, the regulator will leave the firm with no profits. The firm can do no worse than this, so there is an ex-ante constraint that $r(k) - \rho k \geq 0$. Since $r(\underline{k}) = 0$ from equation (3′), this implies $\underline{k} = 0$, so that the lower end of the distribution represents no *ex ante* investment in cost reducing activity. Clearly if the firm were to choose a $k > k^*$ with positive probability, there would be an advantage to reduce $\bar{k}$ by dk leading to a cost saving of more than ρdk. The only efficiency benefit from not doing so would be if there were additional efficiency savings from a reduction in falsification costs, but as there is no cost padding at $\bar{k}$, there are no such benefits and it is better to reduce $\bar{k}$ to k^*. □

That $\underline{k} = 0$ is a manifestation of the hold-up problem for the firm's investment in cost reduction. Once the firm has invested in cost reducing activity at the pre-contractual stage, the regulator can extract the entire rent from the highest cost firm and thus in order to have non-negative profits ex ante, no extra investment in cost reduction must be an option for the firm. Thus we can conclude from the participation constraint of equation (3′) that the rent function satisfies $r(k) = \rho k$.[24] The upper endpoint of the distribution can never be inefficiently high because of the efficiency at the top property and hence at the upper endpoint $\bar{k} = k^*$ and $q = q^*$.

We can now use our definition of equilibrium to show that the equilibrium distribution has no mass points. Using equation (9) and integrating by parts the regulator's virtual surplus given by equation (1″) can be rewritten as

$$\int_0^{k^*} \omega(C,k,q)\,\mathrm{d}F(k) - (1+\lambda)\rho \int_0^{k^*} (1-F(k))\,\mathrm{d}k. \tag{10}$$

The virtual surplus then consists of two terms. The first term is the expected net social benefit. The second term is the expected rent paid to the firm weighted by the shadow cost of public funds $(1+\lambda)$. With this observation it is possible to prove the following.

PROPOSITION 4: *In any equilibrium in which the firm adopts a mixed strategy there are no mass points.*

Proposition 4 shows that $\alpha_d = 0$ so that the equilibrium distribution is continuous. The idea of the proof is to consider a distribution with mass points and to show that a change in the distribution that smooths out the discontinuity by appropriately shifting the distribution function whilst keeping the original contract can increase the virtual surplus. Clearly then changing the contract to the optimum for the new distribution will increase the virtual surplus further or at least not decrease it. It is clear from equation (10) that a change to a stochastically dominant distribution increases expected social capital costs but so too will expected net social benefit if $\omega(C(k),k,q(k))$ is increasing in k. The proof of Proposition 4 works by showing that on any continuous part of the distribution the net social benefit function ω increases in k at a rate greater than $(1+\lambda)\rho$, so that an appropriate change in the distribution increases net social benefit by more than the social capital costs and hence will increase the virtual surplus.

[24] Since the rent just covers the investment cost it may be viewed as a rent to the quasi-fixed factor at the contractual stage rather than an information rent.

3.5. *Comparative Statics*

The optimum contract Δ and equilibrium distribution $F(k)$ will be determined by equations (9), (2′), (7′) and (8′) together with equation (2′) and the endpoint conditions for the distribution. This system of equations is in general quite difficult to analyze so to illustrate how the distribution and contract are simultaneously determined in equilibrium consider a procurement variant of the model where output $q \in \{0,1\}$ and the value of the project is fixed at V. Writing $g(k)$ for $g(k,1)$ equation (9) is rewritten as $-(1-\phi'(C-g(k)))g_k(k) = \rho$ and this can be used to determine $C(k)$ directly. From the implicit function theorem $C(k)$ is both continuous and differentiable and decreasing.[25] Supposing that the distribution function $F(k)$ has a density $f(k)$, the hazard rate function[26] $h(k) = f(k)/(1-F(k)$ is determined by equation (7′) and is given by

$$h(k) = -\frac{\lambda}{1+\lambda}\frac{\phi''(C(k)-g(k))g_k(k)}{\phi'(C(k)-g(k))}. \tag{11}$$

Letting $\upsilon(C(k),k) = g(k)+\phi(C(k)-g(k))$, and using $\mathrm{E}[r(k)] = \rho\mathrm{E}[k]$, expected welfare is given by $V-(1+\lambda)(\mathrm{E}[\upsilon(C(k),k)]+\rho\mathrm{E}[k])$.

As a first comparative static exercise we shall consider how the solution changes as the shadow cost of public funds λ changes. Suppose that the shadow cost increases from λ to λ'. This leads to an increase in the hazard rate function. Abusing notation slightly we have from equation (11) that $h_{\lambda'}(k) > h_{\lambda}(k)$ for all k so that the monotone hazard rate condition is satisfied. Integrating the hazard rate therefore shows that the distribution generated with λ *first-order stochastically dominates* the distribution generated with λ'. That is the distribution with λ' puts more weight on low values of k (higher costs) than does the distribution generated with λ. Hence $F_{\lambda'}(k) > F_{\lambda}(k)$ for $k \in (0,k^*)$. Since $C(k)$ is decreasing in k from equation (9), first-order stochastic dominance implies that expected costs are higher with the distribution generated with λ' than the one generated with shadow cost λ. We write this as $\mathrm{E}_{\lambda'}[C(k)] > \mathrm{E}_{\lambda}[C(k)]$ where the subscript indicates that the distribution over which the expectation is taken depends on λ.[27] Likewise, since $x(k)$ is decreasing in k, a higher value of the shadow cost of public funds will be associated with higher expected cost padding. Using equation (9) $\upsilon(C(k),k)+\rho k$ is decreasing in k since $C(k)$ is decreasing in k, $\upsilon_k(C,k) = -\rho$ and $\upsilon_C(C,k) > 0$. Therefore

[25] Hence the second-order condition for the incentive constraint is satisfied.

[26] The hazard rate function is the derivative of the log-survivor function $H(k) = -\log_e(1-F(k))$.

[27] The cost function $C(k)$ is independent of λ.

$(1+\lambda')\mathrm{E}_{\lambda'}[\upsilon(C(k),k)+\rho k] > (1+\lambda)\mathrm{E}_{\lambda}[\upsilon(C(k),k)+\rho k]$. Thus expected welfare falls as the shadow cost of public funds rises.

Now suppose that Assumption 3′ holds and consider a change in the falsification cost parameter α. Let $x_\alpha(k)$ denote cost padding with parameter α and $x_{\alpha'}(k)$ denote cost padding with parameter α'. From equation (9) $x_{\alpha'}(k) = \delta x_\alpha(k)$ where $\delta = (\alpha/\alpha')^{(1/(\beta-1))}$. With an increase in α to α', and as $\delta < 1$, the change will lower the amount of cost padding for each given value of $k < k^*$. This is the direct effect on cost padding of an increase in falsification costs. However, there is an indirect effect on cost padding as the distribution of investment changes. Equation (11) shows that $h_{\alpha'}(k) = (1/\delta)h_\alpha(k)$. Thus for $\alpha' > \alpha$ the distribution with parameter α first-order stochastically dominates the distribution with parameter α' and so $F_{\alpha'}(k) > F_\alpha(k)$ for $k \in (0,k^*)$. In particular integrating the hazard functions shows that $F_{\alpha'}(k) = 1-(1-F_\alpha(k))^{(1/\delta)}$. Hence although cost padding falls for any given k the distribution changes to put more weight on lower values of k and since $x(k)$ is decreasing in k, this will increase expected cost padding. It is however, shown in the next Proposition that the direct effect dominates and the net effect of the increase in the falsification cost is to reduce expected levels of cost padding. Proposition 5 also shows that expected falsification costs $\mathrm{E}[\phi(x)]$ decrease in α and that expected costs $\mathrm{E}[g(k)+\rho k]$ increase in α.

PROPOSITION 5: *A rise in the falsification cost from α to α' reduces expected cost padding: $E_{\alpha'}[x_{\alpha'}] < E_\alpha[x_\alpha]$; reduces falsification costs: $E_{\alpha'}[\phi(x_{\alpha'})] < E_\alpha[\phi(x_\alpha)]$; and increases costs net of falsification costs: $E_{\alpha'}[g(k)+\rho k] > E_\alpha[g(k)+\rho k]$.*

The net effect on welfare is generally ambiguous although expected falsification costs fall with an increase in α, the leftward shift of the distribution will increase expected real costs.[28]

As another comparison consider a change in the cost of capital ρ. Since the upper endpoint of the distribution k^* depends on ρ an increase in ρ to ρ' will lower the upper endpoint of the distribution from $k^*(\rho)$ to $k^*(\rho')$. Also from equation (9) the increase in ρ will reduce cost padding for each $k > 0$. Thus the direct effect of an increase in ρ is to reduce cost padding. The indirect effect of changes in ρ through the change in x is again to shift the distribution function leftward so the distribution with parameter ρ first-order stochastically dominates the distribution with parameter ρ'. However, the net effect on cost padding and welfare will in general depend on parameters of the cost function. The effect on the expected capital of the firm is unambiguously negative as the distribution has shifted leftward. However, the effect

[28] A simple example of the effect of a change in α on welfare is given below.

on rent is ambiguous as although capital is decreased, it has higher value as ρ has increased. Thus the effect on welfare will again be generally ambiguous.

Finally we give a simple example to illustrate how the solution depends on the cost of falsification α and the cost of capital ρ.

EXAMPLE 3: $g(k) = 2(2-\sqrt{k})$; $\phi(x) = (\alpha/2)x^2$, $\lambda = 1$. Equation (9) shows that cost padding is $x(k) = (1-\rho\sqrt{k})/\alpha$. The distribution function is defined on $[0,k^*]$ where $k^* = 1/\rho^2$.[29] Equation (11) determines the hazard rate function $h(k)$ and integrating gives the distribution function $F(k) = 1-(1-\rho\sqrt{k})^{\alpha/\rho}$. The transfer function is $t(k) = (\rho k(2\alpha+\rho)-1)/(2\alpha)$, alternatively, as a function of C, $t(C) = \left((4(1+2\alpha)\rho-1)-(1+4\alpha)\rho C+(1/2)\alpha\rho C^2\right)/(2\alpha+\rho)$. Expected cost padding is $\mathrm{E}[x] = 1/(\alpha+\rho)$, expected cost is $\mathrm{E}[g] = 4-(2/(\alpha+\rho))$, expected falsification cost is $\mathrm{E}[\phi(x)] = 1/(2(\alpha+2\rho))$ and therefore the expected rent is $\mathrm{E}[r] = \rho\mathrm{E}[k] = 2\rho/((\alpha+\rho)(\alpha+2\rho))$. It can be seen that in this example expected cost padding is decreasing in α and ρ so that the direct effect dominates. Expected costs increase in α and ρ and expected welfare decreases with α and ρ. The expected rent decreases with α but is non-monotonic in ρ. Expected welfare is $\mathrm{E}[W] = V-8+(3/(\alpha+2\rho))$. Table 1 gives some values for α and ρ.

TABLE 1: SOME TABULATED VALUES

	$\rho=1,\alpha=1$	$\rho=1,\alpha=2$	$\rho=\frac{1}{2},\alpha=1$	$\rho=\frac{1}{2},\alpha=2$
$\mathrm{E}[x]$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{5}$
$\mathrm{E}[\phi]$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{6}$
$\mathrm{E}[g]$	3	$\frac{10}{3}$	$\frac{8}{3}$	$\frac{16}{5}$
$\mathrm{E}[r]$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{15}$
$\mathrm{E}[W]$	$V-7$	$V-\frac{29}{4}$	$V-\frac{13}{2}$	$V-7$

3.6. Endogenous versus Exogenous Distributions

The solution when the distribution of pre-contractual investment choices is determined endogenously might be quite different from the solution when the distribution of investment choices is exogenously given. This section will illustrate some of these differences for the procurement problem.

[29] It therefore follows that $\phi'(x) = \alpha x < 1$ for all $k \in [0,k^*]$.

Consider the case when cost padding costs are given by Assumption 3′. Fix some level of α, say $\bar{\alpha}$, and suppose that in the exogenous case the hazard rate function $h^e(k)$ is set equal to the hazard rate in the endogenous case with $\alpha = \bar{\alpha}$. With a fixed distribution, cost padding is determined directly from equation (11) so that the amount of cost padding is independent of α. Denote the level of cost padding in this exogenous case be $x^e(k)$. From Proposition 5, cost padding $x(k)$ in the endogenous case is inversely related to α. It is easy to check that $x(k) = \delta x^e(k)$ and $h(k) = (1/\delta)h^e(k)$ where $\delta = (\bar{\alpha}/\alpha)^{(1/(\beta-1))}$. Thus for $\alpha < \bar{\alpha}$ we have $x(k) > x^e(k)$ for each value of k and cost padding is raised relative to the exogenous case. However, since $h(k) < h^e(k)$ for $\alpha < \bar{\alpha}$, integration shows that $(1-F(k)) = (1-F^e(k))^{(1/\delta)}$ so that the distribution function shifts rightward and the endogenous distribution first-order stochastically dominates the exogenous distribution. Nevertheless using Proposition 5 it follows that the direct effect dominates and $\mathrm{E}[x(k)] \gtreqless \mathrm{E}[x^e(k)]$ as $\bar{\alpha} \gtreqless \alpha$.

In the exogenous case expected falsification costs $\mathrm{E}[\phi(x^e(k))]$ increase linearly in α whereas Proposition 5 shows that as the distribution adjusts to an increase in α by reducing cost padding, the expected falsification costs $\mathrm{E}[\phi(x(k))]$ decrease even though the distribution shifts rightward. Likewise in the exogenous case an increase in α leaves true costs unchanged whereas the distribution shifts to the right and thus raises expected costs in the endogenous case. In both the exogenous and endogenous cases the expected rent falls with an increase in α but for rather different reasons. In the exogenous case an increase in α raises marginal falsification costs and thus reduces the increase in rent that is offered to lower cost types. In the endogenous case rent just covers capital costs but the shift in the distribution towards lower investment reduces the expected rent.

In both cases the effect of a change in α on welfare is generally ambiguous but the effect may be quite different in the two cases.[30] Figure 1 provides an illustration and compares Example 2, where the distribution function is exogenously given by $F(k) = \sqrt{k}$ on $[0,1]$, with Example 3 (setting $\rho = 1$), where the distribution function is endogenously determined by $F(k) = 1-(1-\sqrt{k})^{\alpha}$ on $[0,1]$. For $\alpha = 1$ the two distribution functions are equal and the solutions are exactly the same for this value. However, as α is lowered below one the two solutions will diverge.[31] The figure plots expected cost padding, expected total costs, expected falsification costs and expected welfare.[32] The darker line depicts the solution in the exogenous case and

[30] Remember that expected welfare is given by $V-(1+\lambda)(\mathrm{E}[\upsilon(C(k),k)])-\lambda\mathrm{E}[r(k)]-\rho\mathrm{E}[k]$.

[31] The solution in Example 2 is only defined for $\alpha < 1$.

[32] The value of the project has been arbitrarily set at $V = 8$ which is sufficient for the project to be worthwhile.

the lighter line the solution when the distribution adjusts to a change in α. As can be seen from the diagram expected cost padding and expected falsification costs are higher in the endogenous case for $\alpha < 1$ (Figure 1(a) and Figure 1(c)). However, real costs are lower in the endogenous case as shown in Figure 1(b). The net effect on welfare is illustrated in Figure 1(d). Welfare is higher in the endogenous case but is decreasing in α whereas in the exogenous case welfare increases with α.

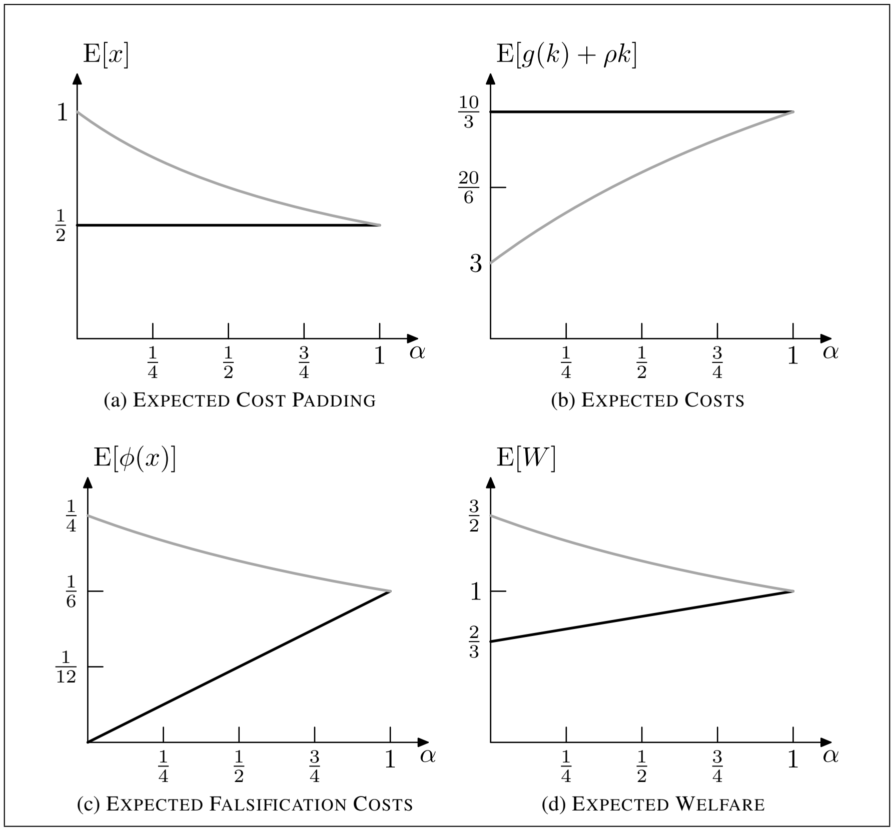

FIGURE 1: COMPARISONS OF EXAMPLES 2 AND 3 FOR $\rho = 1$ AND $V = 8$.

Figure 1 shows that the distinction between the case of a fixed distribution and the endogenous case where the distribution shifts is important for welfare and hence has potential policy implications. For example, consider a policy that intends to raise falsification costs, through improved accountancy regulations or closer monitoring. This would lead to an increase in welfare if the distribution is fixed. However, in the endogenous case, although this will lead to a fall in falsification costs, it will also lead to a shift in the distribution toward lower investment and higher costs and overall, in this example, reduce welfare.

4. CONCLUSION

This paper has extended the standard model of regulated monopolies to allow for both cost padding and incentives for cost reduction. It has been assumed that the cost reducing activity is undertaken at a pre-contractual stage and any cost padding is undertaken post-contractually. It is shown how some cost padding will be tolerated in optimal regulatory contracts. It also induces a move away from Ramsey pricing and implies weaker price regulation than without cost padding in the case where the project produces a private good. By allowing firms to undertake a pre-contractual investment in cost reduction, the distribution of cost types is derived endogenously and therefore the properties of the cost-reimbursement contract depend only on the fundamental technology and preference parameters of the model and do not depend on arbitrary assumptions about the distribution of types. With pre-contractual investment determined endogenously investment in cost reduction is distorted below its cost minimizing level. The paper has also illustrated the importance of studying this pre-contractual choice of firms for welfare and policy implications.

APPENDIX

LEMMA 2: *Given a distribution for the regulator's beliefs F^e then for any two isolated but adjacent values in the support, the downward incentive compatibility constraints bind at the optimum contract.*

PROOF: Consider two adjacent values k^+ and k^- where $k^+ > k^-$. Suppose, contrary to the assertion in the lemma, that $r(k^+,k^+) > r(k^-,k^+)$. That is suppose

$$t(k^+)+C(k^+)-\upsilon(C(k^+),k^+,q(k^+)) > t(k^-)+C(k^-)-\upsilon(C(k^-),k^+,q(k^-)).$$

From the incentive compatibility conditions of equation (2) it is easy to check that $r(\hat{k}) > r(k^+)$ for all $\hat{k} > k^+$ as $\upsilon(C,k,q)$ is decreasing in k. Also $r(\tilde{k},k^+) > r(\tilde{k},\hat{k})$ again since $\upsilon(C,k,q)$ is decreasing in k. This implies that the local downward incentive compatibility constraints imply the constraints for all higher values of k are also satisfied. Now suppose the regulator lowers $t(k^+)$ so that $r(k^+,k^+) = r(k^-,k^+)$ and lowers $t(\hat{k})$ by the same amount for all $\hat{k} > k^+$. This change will not affect incentive compatibility since we know that the constraints for higher k remain satisfied and it otherwise either relaxes the constraints or leaves them unchanged. Such a change increases the regulator's welfare and so in any optimum the adjacent downward incentive constraints bind. □

LEMMA 3: *If the regulator's beliefs are consistent then the set $\mathscr{S}^e$ is an interval.*

PROOF: The support of a distribution is the minimal closed set whose complement has probability zero. Suppose then, contrary to the lemma, that the support $\mathscr{S}^e$ is not an interval. Then there are values k, k' and k'' with $k' < k < k''$ with $k', k'' \in \mathscr{S}^e$ and $k \notin \mathscr{S}^e$. Let $k^- = \sup\{k' \in \mathscr{S}^e \colon k' < k\}$ and $k^+ = \inf\{k'' \in \mathscr{S}^e \colon k'' > k\}$. From Lemma 2 in an optimal contract the local downward incentive compatibility constraint between k^+ and k^- binds. We shall now show that given such an optimum contract, the firm can always deviate and raise profits by choosing some k between any k^- and k^+ and reporting k^-. Let π^- denote profit, t^- denote the transfer and so on when k^- is chosen. Then

$$\begin{aligned} \pi^- &= t^- + C^- - \upsilon(C^-, k^-, q^-) - \rho k^- \\ \pi^+ &= t^- + C^- - \upsilon(C^-, k^+, q^-) - \rho k^+ \end{aligned}$$

where the second equation uses the fact that the downward incentive compatibility constraint binds. Now suppose that the firm chooses some $k \in (k^-, k^+)$. Let $\pi(k^-, k)$ denote the level of profits from choosing k but reporting k^-. To be an equilibrium the original contract must remain incentive compatible, $\pi(k^-, k) < \pi^- = \pi^+$. Using the previous equations

$$\begin{aligned} \pi(k^-, k) - \pi^- &= \upsilon(C^-, k^-, q^-) - \upsilon(C^-, k, q^-) - \rho(k - k^-) \\ \pi(k^-, k) - \pi^+ &= \upsilon(C^-, k^+, q^-) - \upsilon(C^-, k, q^-) + \rho(k^+ - k). \end{aligned}$$

Therefore if the firm is not to gain from deviation the following conditions must hold

$$\begin{aligned} \upsilon(C^-, k^-, q^-) - \upsilon(C^-, k, q^-) &< \rho(k - k^-) \\ \upsilon(C^-, k, q^-) - \upsilon(C^-, k^+, q^-) &> \rho(k^+ - k). \end{aligned}$$

Taking any k such that $k^- < k \leq \frac{1}{2}(k^- + k^+)$ and given that υ is strictly decreasing this implies that

$$\begin{aligned} 0 < \upsilon(C^-, k^-, q^-) - \upsilon(C^-, k, q^-) < \rho(k - k^-) \leq \rho(k^+ - k) \\ < \upsilon(C^-, k, q^-) - \upsilon(C^-, k^+, q^-). \end{aligned}$$

But for $k = \frac{1}{2}(k^- + k^+)$ this contradicts the strict convexity of υ in k which follows from Assumptions 2 and 3. Thus if $\mathscr{S}^e$ is not an interval the firm will always gain by deviating and choosing some investment level not in $\mathscr{S}^e$ between the two adjacent points in $\mathscr{S}^e$ and reporting the lower investment level (higher cost). □

LEMMA 4: *Necessary and sufficient conditions for incentive compatibility are*

(i) $r(k) = r(\underline{k}) - \int_{\underline{k}}^{k} \upsilon_k(C(\kappa), \kappa, q(\kappa))\,\mathrm{d}\kappa$

(ii) $-\int_{k}^{k'} (\upsilon_k(C(k'), \kappa, q(k')) - \upsilon_k(C(\kappa), \kappa, q(\kappa)))\,\mathrm{d}\kappa \geq 0 \quad \forall k$ *and* k'.

PROOF: We shall first establish necessity and show that the incentive compatibility constraints given in equation (2) imply the two conditions of the lemma. As $r(k) = t(k) + C(k) - \upsilon(C(k), k, q(k))$ and $r(k') = t(k') + C(k') - \upsilon(C(k'), k', q(k'))$, the incentive compatibility constraints of equation (2) give

$$\begin{aligned} r(k) &\geq t(k') + C(k') - \upsilon(C(k'), k, q(k')) \\ r(k') &\geq t(k) + C(k) - \upsilon(C(k), k', q(k)). \end{aligned}$$

Combining the definitions with the incentive constraints gives

$$\begin{aligned} -\big(\upsilon(C(k'), k', q(k')) - \upsilon(C(k'), k, q(k'))\big) &\geq r(k') - r(k) \\ &\geq -\big(\upsilon(C(k), k', q(k)) - \upsilon(C(k), k, q(k))\big) \end{aligned}$$

and hence by the continuity of $\upsilon(C, k, q)$, the connectedness of the sets $\mathscr{C}$ (given the continuity of g) and $\mathscr{Q}$, the intermediate value theorem guarantees that this can be satisfied if and only if there are values $\bar{C}(k', k)$ and $\bar{q}(k', k)$ such that

$$\begin{aligned} r(k') - r(k) &= -\big(\upsilon(\bar{C}(k', k), k', \bar{q}(k', k)) - \upsilon(\bar{C}(k', k), k, \bar{q}(k', k))\big) \\ &= -\int_{k}^{k'} \upsilon_k(\bar{C}(k', k), \kappa, \bar{q}(k', k))\,\mathrm{d}\kappa. \end{aligned}$$

Suppose without loss of generality that $k' > k$. Then using the Mean Value Theorem there is some $\hat{k} \in [k, k']$ such that

$$\frac{r(k') - r(k) k' - k}{=} \upsilon_k(\bar{C}(k', k), \hat{k}, \bar{q}(k', k)). \tag{A.1}$$

Then by compactness we can find convergent subsequences such that $\bar{C}(k', k) \to \bar{C}(k)$ and $\bar{q}(k', k) \to \bar{q}(k)$ as $k' \to k$ so that taking the limit as $k' \to k$ gives

$$\dot{r}(k) = -\upsilon_k(\bar{C}(k), k, \bar{q}(k))$$

wherever this is defined. Then using equation (A.1) and taking the absolute value shows that

$$|r(k') - r(k)| \leq \upsilon_k(\bar{C}(k',k), \hat{k}, \bar{q}(k',k)|k'-k| \leq M|k'-k|$$

for some finite number M since υ_k is assumed continuous on a closed and bounded interval and hence itself bounded. Hence $r(k)$ is Lipschitz continuous and hence absolutely continuous. Thus we can use the fundamental theorem of calculus and write

$$r(k) = r(\underline{k}) + \int_{\underline{k}}^{k} \dot{r}(\kappa)\,\mathrm{d}\kappa. \tag{A.2}$$

Equally using the definition of $r(k',k)$,

$$r(k',k) = r(k') + \upsilon((C(k'),k',q(k')) - \upsilon(C(k'),k,q(k')).$$

Hence using equation (A.2) we have

$$\begin{aligned} r(k) - r(k',k) &= r(k) - r(k') - \int_{k}^{k'} \upsilon_k(C(k'),\kappa,q(k'))\,\mathrm{d}\kappa \\ &\quad - \int_{\underline{k}}^{k} \upsilon_k(C(\kappa),\kappa,q(\kappa))\,\mathrm{d}\kappa + \int_{\underline{k}}^{k'} \upsilon_k(C(\kappa),\kappa,q(\kappa))\,\mathrm{d}\kappa \\ &\quad - \int_{k}^{k'} \upsilon_k(C(k'),\kappa,q(k'))\,\mathrm{d}\kappa \\ &= -\int_{k}^{k'} \big(\upsilon_k(C(k'),\kappa,q(k')) - \upsilon_k(C(\kappa),\kappa,q(\kappa))\big)\,\mathrm{d}\kappa \end{aligned}$$

Then since incentive compatibility requires $r(k) - r(k',k) \geq 0$ for all k and k' we have the second condition of the Lemma. Now consider sufficiency. It is obvious from the above equation that if the second condition of the Lemma is satisfied then $r(k) - r(k',k) \geq 0$ for all k and k' and hence it is incentive compatible. Equally if Assumption 4 is satisfied then $r(k)$ is incentive compatible if it satisfies the integral condition. From the single crossing property it is always possible to find some (C_1,q_1) and (C_2,q_2) such that

$$\upsilon_k(C_1,k,q_1) \leq \upsilon_k(C,k,q) \leq \upsilon_k(C_2,k,q_2)$$

for all k, C, q. Then since the integral condition is satisfied

$$\int_k^{k'} \upsilon_k(C_1, \kappa, q_1)\,\mathrm{d}\kappa \le \int_k^{k'} \upsilon_k(C, \kappa, q)\,\mathrm{d}\kappa \le \int_k^{k'} \upsilon_k(C_2, \kappa, q_2)\,\mathrm{d}\kappa$$

or

$$\upsilon(C_1, k', q_1) - \upsilon(C_1, k, q_1) \le r(k') - r(k) \le \upsilon(C_2, k', q_2) - \upsilon(C_2, k, q_2)$$

where C is chosen equal to $\bar{C}(k', k)$ and $q = \bar{q}(k', k)$.

$$r(k') - r(k) = \upsilon(\bar{C}(k', k), k', \bar{q}(k', k)) - \upsilon(\bar{C}(k', k), k, \bar{q}(k', k)).$$

so that reversing the argument given above $r(k)$ is incentive compatible. □

PROPOSITION 4: *In any equilibrium in which the firm adopts a mixed strategy there are no mass points*

PROOF: Suppose there is a mass point at k_i. Let the mass at this point be f_d where $F(k_i-) + f_d = F(k_i)$. We shall define a new distribution $F_\varepsilon(k)$ such that

$$F_\varepsilon(k) = \begin{cases} F(k) + \frac{f_d}{\varepsilon}(k - (k_i + \varepsilon)) & \text{if } k \in [k_i, k_i + \varepsilon] \\ F(k) & \text{otherwise} \end{cases}$$

where $\varepsilon > 0$. We can choose ε small so that $F(k)$ is continuous on $[k, k+\varepsilon]$. Note that $F_\varepsilon(k_i) = F(k_i) - f_d = F(k_i-)$ and $F_\varepsilon(k_i + \varepsilon) = F(k_i + \varepsilon)$. Hence it can be concluded that $(1 - F_\varepsilon(k)) - (1 - F(k)) = -(f_d/\varepsilon)(k - (k_i + \varepsilon))$ on $[k, k+\varepsilon]$. Also since F is continuous on $(k_i, k_i + \varepsilon]$ we have the differential $dF_\varepsilon(k) = dF(k) + (f_d/\varepsilon)$ on this interval. Consider then a change in the distribution from $F(k)$ to $F_\varepsilon(k)$, but keeping the contract unchanged. The contract remains incentive compatible but may no longer be optimal. Using equation (10) the effect of such a change in distribution on total welfare is

$$\int_{k_i}^{k_i+\varepsilon} \omega(C(k), k, q(k))\,(\mathrm{d}F_\varepsilon(k) - \mathrm{d}F(k)) - \omega(C(k_i), k_i, q(k_i)) f_d \\ - (1+\lambda)\rho \int_{k_i}^{k_i+\varepsilon} ((1 - F_\varepsilon(k)) - (1 - F(k)))\,\mathrm{d}k.$$

Using the definition for F_ε this can be rewritten as

$$\frac{f_d}{\varepsilon}\left(\int_{k_i}^{k_i+\varepsilon} \{\omega(C(k), k, q(k)) - \omega(C(k_i), k_i, q(k_i)) + (1+\lambda)\rho(k - (k_i + \varepsilon))\}\,\mathrm{d}k\right).$$

For notational simplicity write $C = C(k)$, $C_i = C(k_i)$ and so on. Decomposing the change in ω we have

$$\omega(C,k,q) - \omega(C_i,k_i,q_i) = (\omega(C,k,q) - \omega(C_i,k,q_i)) + (\omega(C_i,k,q_i) - \omega(C_i,k_i,q_i)).$$

By Assumption 5, ω is strictly concave in C and q and hence since it is differentiable

$$\omega(C,k,q) - \omega(C_i,k,q_i) > -\omega_C(C,k,q)(C_i - C) - \omega_q(C,k,q)(q_i - q).$$

Then using the first-order conditions (7) and (8) we have

$$-\omega_C(C,k,q)(C_i - C) - \omega_q(C,k,q)(q_i - q) = \varphi(k)(\upsilon_{kC}(C,k,q)(C_i - C) + \upsilon_{kq}(C,k,q)(q_i - q))$$

for some constant $\varphi(k) > 0$. Using the strict concavity and differentiability of υ_k

$$\upsilon_{kC}(C,k,q)(C_i - C) + \upsilon_{kq}(C,k,q)(q_i - q) > \upsilon_k(C_i,k,q_i) - \upsilon_k(C,k,q).$$

However by equation (9) $\upsilon_k(C,k,q) = \upsilon_k(C_i,k_i,q_i)$ and hence

$$\upsilon_k(C_i,k,q_i) - \upsilon_k(C,k,q) = \upsilon_k(C_i,k,q_i) - \upsilon_k(C_i,k_i,q_i).$$

Hence

$$\omega(C,k,q) - \omega(C_i,k,q_i) > \varphi(k)(\upsilon_k(C_i,k,q_i) - \upsilon_k(C_i,k_i,q_i)).$$

Since we also have

$$\omega(C_i,k,q_i) - \omega(C_i,k_i,q_i) = -(1+\lambda)(\upsilon(C_i,k,q_i) - \upsilon(C_i,k_i,q_i)).$$

Putting these two conditions together we therefore have that

$$\omega(C,k,q) - \omega(C_i,k_i,q_i) > \varphi(k)(\upsilon_k(C_i,k,q_i) - \upsilon_k(C_i,k_i,q_i)) - (1+\lambda)(\upsilon(C_i,k,q_i) - \upsilon(C_i,k_i,q_i)).$$

Then using the mean value theorem to find values k^1 and k^2 between k_i and k so that

$$\omega(C,k,q) - \omega(C_i,k_i,q_i) > -(1+\lambda)\upsilon_k(C_i,k_i,q_i)(k - k_i) + \varphi(k)\upsilon_{kk}(C_i,k^1,q_i)(k - k_i) - (1+\lambda)(\upsilon_k(C_i,k^2,q_i) - \upsilon_k(C_i,k_i,q_i))(k - k_i)$$

where $(1+\lambda)\upsilon_k(C_i,k_i,q_i)(k-k_i)$ has been added and subtracted to the right-hand-side of the equation. Applying the mean value theorem once again we can find a value k^3 between k^2 and k_i and using $-\upsilon_k(C_i,k_i,q_i)=\rho$ the inequality becomes

$$\begin{aligned}\omega(C,k,q)-\omega(C_i,k_i,q_i) >&(1+\lambda)\rho(k-k_i)+\varphi(k)\upsilon_{kk}(C_i,k^1,q_i)(k-k_i)\\ &-(1+\lambda)\upsilon_{kk}(C_i,k^3,q_i)(k-k_i)(k^2-k_i)\end{aligned}$$

As $k>k_i$, diving through by $k-k_i$ shows that we can find some $\varepsilon>0$ such that with $k-k_i<\varepsilon$,

$$\omega(C,k,q)-\omega(C_i,k_i,q_i)>(1+\lambda)\rho(k-k_i).$$

Substituting into the expression for total welfare and integrating then shows that the change in the welfare is positive. Since this improvement is achieved without changing the contract as the distribution is changed it is clear that the optimal contract in the new distribution cannot lower welfare and hence the original distribution could not have been an equilibrium. Finally note that the above arguments rules out mass points by changing the distribution to the right of the mass point and therefore does not work if there is a mass point at k^*. However if there is a mass of f_* at k^* a similar argument works be defining a distribution

$$F_\varepsilon(k)=\begin{cases}F(k)+\frac{f_*}{\varepsilon}(k-(k^*-\varepsilon)) & \text{if } k\in[k^*-\varepsilon,k^*]\\ F(k) & \text{otherwise}\end{cases}$$

which smooths out the mass point to the left. Reapplying the same argument as above also shows that such a change increases welfare and therefore the original distribution with mass at k^* could not have been an equilibrium. □

PROPOSITION 5: *A rise in the falsification cost from α to α' reduces expected cost padding: $E_{\alpha'}[x_{\alpha'}]<E_\alpha[x_\alpha]$; reduces falsification costs: $E_{\alpha'}[\phi(x_{\alpha'})]<E_\alpha[\phi(x_\alpha)]$; and increases costs net of falsification costs: $E_{\alpha'}[g(k)+\rho k]>E_\alpha[g(k)+\rho k]$.*

PROOF: (i) $x_{\alpha'}(k)=\delta x_\alpha(k)$, $h_{\alpha'}(k)=(1/\delta)h_\alpha(k)$ where $\delta=(\alpha/\alpha')^{(1/(\beta-1))}$. Integrating $h(k)$ gives $-\log_e(1-F_{\alpha'}(k))=-(1/\delta)\log_e(1-F_\alpha(k))$ and therefore $F_{\alpha'}(k)=1-(1-F_\alpha(k))^{(1/\delta)}$. As there is no cost padding at k^*, $x_{\alpha'}(k^*)=x_\alpha(k^*)=0$ and so integrating by parts expected cost padding is given by

$$\mathrm{E}_{\alpha'}[x_{\alpha'}]=-\int_0^{k^*}x'_{\alpha'}(k)F_{\alpha'}(k)\,\mathrm{d}k=-\int_0^{k^*}x'_\alpha(k)\delta\left(1-(1-F_\alpha(k))^{\frac{1}{\delta}}\right)\mathrm{d}k$$

since $x'_{\alpha'}(k) = \delta x'_{\alpha}(k)$. Likewise

$$\mathrm{E}_{\alpha}[x_{\alpha}] = -\int_0^{k^*} x'_{\alpha}(k) F_{\alpha}(k)\,\mathrm{d}k$$

We shall show that if $\alpha' > \alpha$ then $x'_{\alpha'}(k)F_{\alpha'}(k) > x'_{\alpha}(k)F_{\alpha}(k)$ for each $k < k^*$. Define the function $u(F_{\alpha}) := \delta\left(1-(1-F_{\alpha})^{1/\delta}\right)$ where $F_{\alpha} \in [0,1]$. Then $u(0) = 0$ and $u(1) = \delta$. Using the mean value theorem there is a $\hat{F}_{\alpha} \in (0, F_{\alpha})$ such that we have $u(F_{\alpha}) = u'(\hat{F}_{\alpha})F_{\alpha}$. Hence $u'(\hat{F}_{\alpha}) = (1-\hat{F}_{\alpha})^{(1/\delta)}$ we have $u(F_{\alpha}) \gtreqless F_{\alpha}$ as $\delta \gtreqless 1$. For $\alpha' > \alpha$, $\delta < 1$ and so $\delta\left(1-(1-F_{\alpha})^{1/\delta}\right) < F_{\alpha}$. Hence since $x'_{\alpha}(k) < 0$, we have $\mathrm{E}_{\alpha'}[x_{\alpha'}] < \mathrm{E}_{\alpha}[x_{\alpha}]$.
(ii) We have

$$\begin{aligned}\frac{\mathrm{E}_{\alpha'}[\phi(x_{\alpha'})]}{\alpha} &= -\int_0^{k^*} \left(\frac{\alpha'}{\alpha}\right)(x_{\alpha'}(k))^{\beta-1} x'_{\alpha'}(k) F_{\alpha'}(k)\,\mathrm{d}k \\ &= -\int_0^{k^*} \delta (x_{\alpha}(k))^{\beta-1} x'_{\alpha}(k)\left(1-(1-F_{\alpha}(k))^{\frac{1}{\delta}}\right)\mathrm{d}k\end{aligned}$$

since $x'_{\alpha'}(k) = \delta x'_{\alpha}(k)$ and $\delta = (\alpha/\alpha')^{(1/(\beta-1))}$. Likewise

$$\frac{\mathrm{E}_{\alpha}[\phi(x_{\alpha})]}{\alpha} = -\int_0^{k^*} (x_{\alpha}(k))^{\beta-1} x'_{\alpha}(k) F_{\alpha}(k)\,\mathrm{d}k.$$

From part (i) for $\delta < 1$, $\delta\left(1-(1-F_{\alpha})^{1/\delta}\right) < F_{\alpha}$ and therefore as $x'_{\alpha}(k) < 0$, $\mathrm{E}_{\alpha'}[\phi(x_{\alpha'})] < \mathrm{E}_{\alpha}[\phi(x_{\alpha})]$.
(iii) We have upon integrating by parts

$$\mathrm{E}_{\alpha'}[g(k)+\rho k] - \mathrm{E}_{\alpha}[g(k)+\rho k] = \int_0^{k^*} \left(g'(k)+\rho\right)(F_{\alpha}(k) - F_{\alpha'}(k))\,\mathrm{d}k.$$

Since $g'(k)+\rho < 0$ for $k < k^*$ and $F_{\alpha'}(k) > F_{\alpha}(k)$ for $k \in (0,k^*)$, we have proved that $\mathrm{E}_{\alpha'}[g(k)+\rho k] > \mathrm{E}_{\alpha}[g(k)+\rho k]$ as required. □

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